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RESOLUTION OF CONVOLVED SIGNALS IN BIOMEDICAL APPLICATIONS

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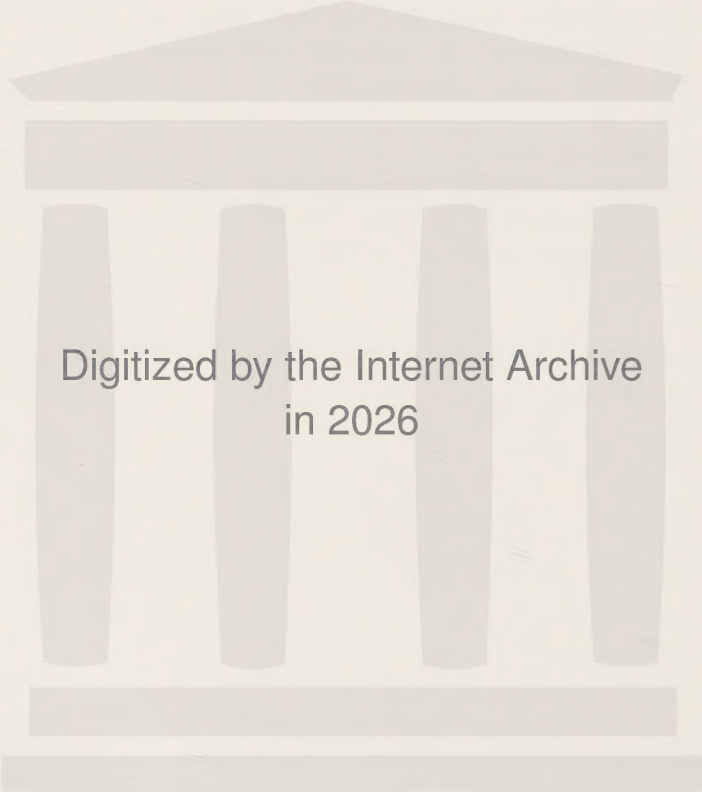
To Josefin



The thesis consists of this summary and the following three papers:

- I. Bengt Mandersson
"DESIGN OF FIR FILTERS FOR DECONVOLUTION"
- II. Bengt Mandersson
"LEAST SQUARES FILTERS USING BANDPASS SUBFILTERS"
Technical Report TR-161, Telecommunication Theory,
University of Lund, November 1981.
- III. Bengt Mandersson
"DIGITAL FILTERING OF ULTRASONIC ECHO SIGNALS"
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In the following in this summary these papers are referred to as I, II and III, respectively.



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INTRODUCTION

Biomedical signals often consist of delayed superposed (interfering) pulses, e.g. in the ultrasonic pulse echo method. The received signal can be modelled as the convolution of a pulse train with the impulse response of a linear system. The estimation of the arrival times of the pulses in the pulse train is then said to be a problem of deconvolution.

Deconvolution problems are often treated with inverse filtering techniques. Inverse filters and filters based on least squares costs have been dealt with by several authors [1,2]. This thesis deals with the resolution of convolved biomedical signals using digital pulse shaping filters with finite impulse responses (FIR).

The application of the pulse shaping technique to deconvolution problems yield filters with "inverse" properties relative to the input signals. This means e.g. that the magnitudes of the spectra of the filter impulse responses must be high where the magnitudes of the input signal spectra are low. This filtering technique is consequently sensitive to noise and to variations of the input signal waveforms. When designing such filters, the trade-off between noise sensitivity and pulse shaping performance must be taken into account. It is then of great importance to have methods for determining filters that are robust in the sense that they are as insensitive as possible to noise and to variations of the input waveforms.

The pulse shaping filters are used in order to resolve superposed signals of different waveforms and to decrease the duration of individual pulses. Examples of these applications are electromyography (EMG) and ultrasound. Part I deals with the design of pulse shaping filters. Iterative solution schemes with steepest descent algorithms are often used for determining filters. Part II deals with the problem of increasing the rate of convergence for steepest descent algorithms. The least squares pulse shaping filter is used in particular to demonstrate the improved rate of convergence achieved by the filter configuration described in Part II. But this method can also be used for all of the FIR filters described in Part I. The axial resolution of the ultrasonic pulse echo method is strongly dependent on the duration of the impulse response of the ultrasonic transducer. Part III describes how axial resolution is improved by using pulse shaping filters.

DESIGNING FIR FILTERS

Pulse shaping filters applied to deconvolution problems are often determined by minimizing the mean squared difference between the filter response to a specified input and some desired pulse shape (least squares filters). However, the solution can be sensitive to the detailed structure of the desired pulse. In many applications it is not always obvious what desired pulse shape to select in order to obtain the most suitable filter. Part I defines "soft" versions of pulse shaping filters. These are also based on minimizing squared differences between the filter responses and desired pulses but with the modification that regions are inserted where the differences do not affect the solution. Both amplitude and temporal regions are examined.

ITERATIVE SOLUTION OF FIR FILTERS

The number of adjustable parameters in the design of FIR filters is quite high. Iterative solution schemes using a steepest descent algorithm are most easy to implement, since they do not need so much computer storage space and the amount of computation in each iteration is low. But the inverse filter technique leads to ill-conditioned minimization problems. It has been pointed out [3] that as the length of the FIR filter is increased the eigenvectors of the correlation matrix become increasingly sinusoidal. Part II describes how the rate of convergence for the steepest descent algorithm can be increased by configuring the FIR filters as sets of bandpass subfilters in parallel. The computations are done in the frequency domain and an FFT routine is used to determine the DFT of the signals. This not only reduces the amount of computation but also suitably relates the temporal properties of the filter to its frequency characteristics.

THE ULTRASONIC PULSE ECHO METHOD

The development of high speed A/D and D/A converters now makes it possible to apply digital processing techniques to ultrasonic signals. In the ultrasonic pulse echo method axial resolution is strongly dependent on the durations of the impulse responses of the ultrasonic transducers. These impulse responses normally have narrow bandpass characteristics. Owing to this it is difficult to derive inverse filters suitable for ultrasonic signals. In Part III, the weighted least squares pulse shaping filter defined in Part I is applied to the ultrasonic signals in order to decrease the duration of the received ultrasonic pulses. The weighted least squares filter is found to be suitable for ultrasonic signals and improved resolution is achieved using this filter. This filter is used both to filter received echoes and to preshape emitted ultrasonic pulses.

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THREE PAPERS ON
RESOLUTION OF CONVOLVED SIGNALS
IN BIOMEDICAL APPLICATIONS

BENGT MANDERSSON
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Abstract This thesis deals with the design of pulse shaping filters with finite impulse response (FIR) applied to the resolution of superposed signals. The practical applications of the filters are on biomedical signals. Part I deals with the design of pulse shaping filters based on various cost functions. Part II deals with iterative solutions of the least squares pulse shaping filter. These problems are often ill-conditioned. By configuring the FIR filter as a set of bandpass subfilters in parallel the rate of convergence can be increased. In Part III pulse shaping filters are applied to ultrasonic signals. It is shown that the axial resolution of ultrasonic pulse echoes is improved by using digital pulse shaping filters.			
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DESIGN OF FIR FILTERS FOR DECONVOLUTION

BENGT MANDERSSON

This part of the thesis is based on material from the three following papers:

- I. Bengt Mandersson
 "Resolution of Superposed Signals with Envelope-Constrained Filters"
 Technical Report TR-106, Telecommunication Theory.
 University of Lund, June 1978.

- II. Bengt Mandersson
 "Some Alternative Criteria in Designing Pulse Shaping Filters"
 Technical Report TR-125, Telecommunication Theory,
 University of Lund, March 1979.

- III. Bengt Mandersson
 "Weighted Least Squares Pulse Shaping Filters"
 Technical Report TR-159, Telecommunication Theory,
 University of Lund, January 1981.

ABSTRACT

This paper deals with the design of FIR filters for the pulse shaping approach to deconvolution problems. Solutions are given for resolving superposed signals of different waveforms and for decreasing the duration of individual pulses. The filters are based on minimizing squared differences between the filter response to a specified input and some desired pulse shape. The algorithm inserts regions around the output pulse in which the differences do not affect the solution. Both amplitude and temporal regions are treated. Applications to biomedical signal processing such as EMG analysis and filtering of ultrasonic echo signals are illustrated.

CONTENTS

I. INTRODUCTION

II. THE DECONVOLUTION APPROACH

III. THE LEAST SQUARES PULSE SHAPING FILTER

IV. THE ENVELOPE-CONSTRAINED FILTER

V. THE SOFT LEAST SQUARES AND SOFT ENVELOPE-CONSTRAINED FILTERS

VI. THE WEIGHTED LEAST SQUARES FILTER

VII. DISCUSSION AND CONCLUSIONS

APPENDIX

REFERENCES

I. INTRODUCTION

Pulse shaping, including inverse filtering, is widely used in many signal processing applications [10], especially in connection with signal deconvolution [3,4,5,6,7]. This paper deals with the resolution of superposed signals as shown in Figs. 1 and 2. The specific applications dealt with here are on biomedical signal processing, where examples of EMG (Electromyography) signals and ultrasonic signals are treated. Deconvolution problems appear in many other applications such as seismic, radar and sonar signals and in speech processing.

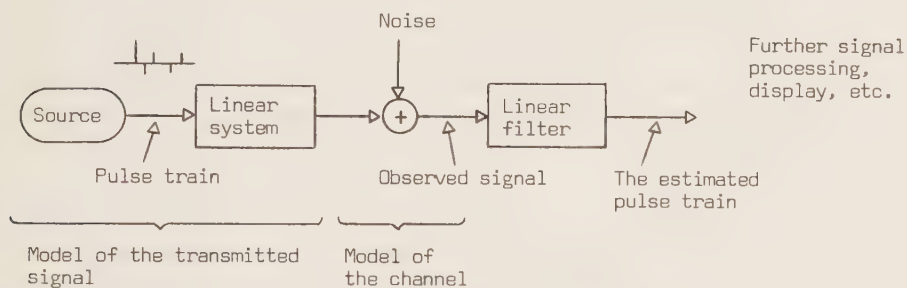


Figure 1. The model of one-source pulse shaping problem.

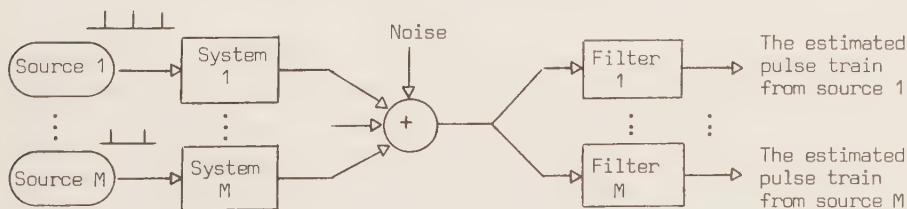


Figure 2. The model of the multi-source problem.

Figure 1 illustrates a model where a single source generates a number of short pulses of varying (and unknown) amplitude. This pulse train is passed through a linear system characterized by its impulse response. It is assumed that this impulse response is known or that it can be estimated. The signal that can be observed is the sum of the original signal and noise with known statistical properties. The problem is to design a linear receiver (filter) such that the receiver output in some sense is a good estimate of the pulse train generated by the source.

The filter is designed to convert the observed signal back to a pulse train approximating the source output (pulse shaping filter or inverse filter). The observed signal is the convolution of the pulse train generated by the source and the impulse response of the linear system following the source. Hence the problem is said to be one of deconvolution or inverse filtering.

The model in Fig. 1 can be applied to ultrasonic pulse measurement. In the ultrasonic pulse echo method a short electrical pulse excites an ultrasonic transducer. As the transmitted acoustical wave passes the surfaces between layers of different acoustical impedance, echo pulses occur. The echo is reflected back to the ultrasonic transducer where it is transformed to an electrical echo signal. The linear system of the model in Fig. 1 corresponds to the impulse response of the ultrasonic transducer, and the pulse train to the echo structure. The observed signal is then the convolution of the transducer impulse response and the model of the echo structure.

Figure 2 illustrates the resolution of signals of different waveforms. In this example, M sources generate pulses of equal amplitude. Then, a linear receiver is designed for each of the sources. An example of superposed signals is found in EMG. The sources correspond to nerve impulses and the linear systems to the so called Motor Unit Potentials (MUP) i.e. the signals generated by the muscle cells. If the degree of the muscle contraction is low, the MUP waveforms can be estimated [15].

In most cases it is not possible to implement the pure inverse filter and approximations have to be used. The inverse filters and filters derived with a least squares approach have been dealt with by several authors, see [3,4]

An envelope-constrained filter is determined in [1] and [2]. It is designed to make the output waveform fit into a pulse shape envelope defined by upper and lower boundaries for a given input pulse.

The approximation approaches described in this paper are the least squares filter, the envelope-constrained filter, the soft least squares filter, the soft envelope-constrained filter and the weighted least squares filter.

In the least squares (LS) approach, a desired output pulse shape (waveform) is first chosen. Then the filters are determined from a cost function which is based upon the square of the error between the actual outputs and the desired outputs. The solution of the LS filter may be sensitive to the choice of desired pulse shape. The envelope-constrained filter (ECF) is based upon upper and lower boundaries of the envelope of the output waveform. The least squares filter and the envelope-constrained filter are described in Sections III and IV, respectively.

For a modified least squares filter called the soft least squares filter (soft LS) a feasible region is defined around the desired pulse shape [17]. The errors are set to zero if the output values lie in this region. The insertion of the feasible region makes the solution of the filter less sensitive to the choice of desired output. Furthermore, the square of the L^2 norm of the filter can be reduced, resulting in a reduction of the output noise power. The soft least squares filter is defined in Section V, which also gives a soft formulation of the envelope-constrained filter (soft ECF).

Section VI deals with another modification of the least squares filter called weighted least squares filter (WLS). The WLS filter describes a wider class of filters in which the least squares filter as well as the matched filter and the pure inverse filter are special cases. The WLS filter is based upon a weighting function (time window). The elements of the error signal (the difference between the desired output waveform and the actual output waveform) are multiplied by different weights for different time intervals. The method outlined in Section VI sets the weights equal to zero in a given interval each side of the main peak of the desired output.

II. THE DECONVOLUTION APPROACH

The pulse shaping problem was outlined in the introduction with reference to specific applications. The basic problem is to estimate some features of the source or sources. However, it is convenient to start by defining the pulse shaping problem as it is understood in this paper. Then, its applications to various problems will be discussed.

Discrete-time signals are assumed. The sample theorem states that a time-continuous signal can be represented by its value at $t = kT_s$, $k = \dots, -1, 0, 1, \dots$ without losing any information, provided the sample rate $f_s = 1/T_s$ is at least twice the highest frequency component of the time-continuous signal. The assumption of discrete-time signals also means that, in the applications, the arrival times of the pulses in the pulse trains are assumed to be multiples of the sampling interval (i.e. integer values). In this case a sample rate higher than that stated by the sample theorem must be used.

The performance criteria used here define cost functions representing the energy of an error signal in a time-limited interval. Then, the optimal filters are found by minimizing the cost functions. The various filters designed in this report differ in the definition of the error signal. The least squares filter is based on a cost function where the error signal is simply the difference between the actual (noiseless) output and a desired output waveform. The least squares filters are used in this section to define the pulse shaping problems.

II.1 The pulse shaping filter

This problem is illustrated by the model in Fig. 1. The filter is designed from the block diagram given in Fig. 3. A source transmits the unit pulse $\delta(t)$. The observed signal is

$$r_s(t) = s(t) + n(t) \quad t \in T_i, T_f \quad (1)$$

The aim is to reconstruct $\delta(t)$. This is described by the desired transmission impulse response $g(t)$ which in some sense is an approximation of $\delta(t)$. The signal is assumed to be time-limited and the observation interval is T_i, T_f . A successful solution will give a small error $e_n(t)$ between the actual output and the desired output.

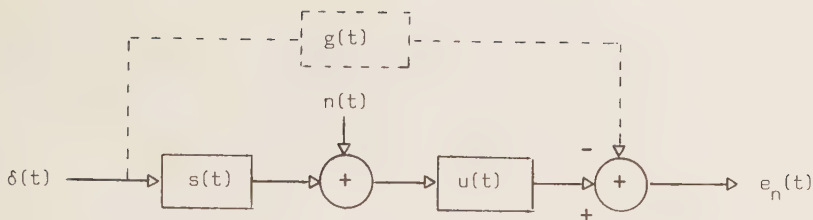


Figure 3. Block diagram of the pulse shaping problem.

The subscript n on the error signal $e_n(t)$ denotes that the input signal (observed signal) is disturbed by noise $n(t)$.

The least squares pulse shaping filter is defined as the filter that minimize the cost function

$$J_{LS} = E \left\{ \sum_{t=T_i}^{T_f} [e_n(t)]^2 \right\} = E \left\{ \sum_{t=T_i}^{T_f} [(s(t)+n(t)) * u(t) - g(t)]^2 \right\} \quad (2)$$

where E is the expected operator.

It is convenient to assume white zero mean noise with the spectral density $N_0/2$ and also that the signal $s(t)$ and the noise are independent. It is also assumed that the signal $s(t)*u(t)$ is completely inside the actual interval T_i, T_f .

Then (2) can be written

$$J_{LS} = \sum_{t=T_i}^{T_f} \left([s(t)*u(t) - g(t)]^2 + E \{ [n(t)*u(t)]^2 \} \right) \quad (3)$$

Equation (3) consists of two components. The first represents the error in the actual noiseless output and the second the contribution of the input noise. The latter can be further rewritten as

$$\begin{aligned} \sum_{t=T_i}^{T_f} E \{ [n(t)*u(t)]^2 \} &= \sum_{t=T_i}^{T_f} \sum_{t_1} \sum_{t_2} E \{ n(t_1)n(t_2) \} u(t-t_1) u(t-t_2) = \\ &= \sum_{t=T_i}^{T_f} \frac{N_0}{2} \sum_{t'} u(t')^2 = (T_f - T_i) \frac{N_0}{2} \sum_{t'} u(t')^2 \quad (4) \end{aligned}$$

Equation (4) represents the noise energy in the time interval $(T_f - T_i)$. The noise energy then depends on the length of this interval. But it would be more appropriate to refer to the same interval as is used to define the signal $s(t)$. Then the influence of the noise is written

$$w_0 \sum_{t'} u(t')^2 \quad (5)$$

with

$$w_0 = \Delta T \cdot \frac{N_0}{2} \quad (6)$$

where ΔT is now the length of the interval over which $s(t)$ is defined. However, in this report w_0 is used as a parameter that gives the filters desirable properties, but w_0 is still referred to as the input noise power. But it is important to keep the expression (6) in mind.

The least squares cost function can now be written

$$J_{LS} = \sum_{t=T_i}^{T_f} [s(t) * u(t) - g(t)]^2 + w_0 \sum_{t'} u(t')^2 \quad (7)$$

The solution of the least squares filter is given in Section III.

II.2 Resolution of superposed signal with pulse shaping filters

The pulse shaping technique can also be applied to the resolution of superposed signals of different waveforms. This multi-source case is shown in Fig. 2. Here it is assumed that all pulses in the pulse train are of equal sign and of equal amplitude. The problem is to "detect" the pulses in the pulse trains, i.e. to estimate their arrival times. It is also assumed that the noise level is low.

The pulse shaping technique is applied by designing a filter $u_i(t)$ for each of the sources i . It is convenient to design the filter corresponding to source 1 first and then drop the subscript of the filter. The application of the pulse shaping filter technique to the resolution of two superposed signals is shown in Fig. 4. The filter $u(t)$ is designed to give a sharp output pulse when the input is $s_1(t)$ and a low output for the input signal $s_2(t)$.

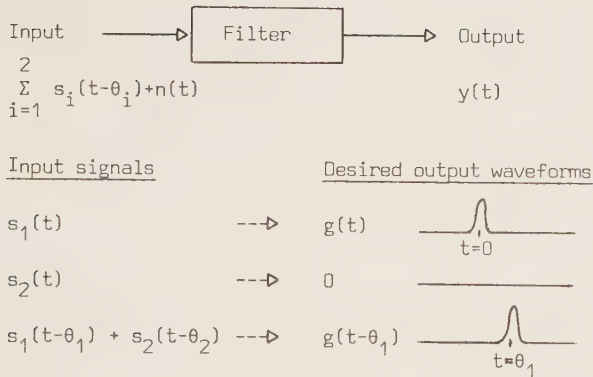


Figure 4. The application of the pulse shaping technique to the resolution of superposed signals.

The least squares cost function is defined as the sum of three components

$$J_{LS} = \sum_{t=T_1}^{T_f} \left\{ [s_1(t) * u(t) - g(t)]^2 + [s_2(t) * u(t)]^2 \right\} + w_0 \sum_t u(t')^2 \quad (8)$$

The first and third components are the same as in the one-source case. The second component gives a low output for the input signal $s_2(t)$. The possibility of designing filters that can be used for resolving superposed signals is, of course, strongly dependent on the waveforms of the signals $s_1(t)$ and $s_2(t)$. The noise level must also be low.

II.3 Discussion of the applications

Two different applications of the pulse shaping filters have been given, see Figs. 1 and 2. In these cases the observed signals are ($M=2$)

$$\text{I. } r(t) = \sum_j a_j s(t-\theta_j) + n(t) \quad (9)$$

$$\text{II. } r(t) = \sum_{j,k} s_1(t-\theta_{1j}) + s_2(t-\theta_{2k}) + n(t) \quad (10)$$

Equation (9) and (10) can be written as

$$\text{I. } r(t) = h(t) * s(t) + n(t) \quad (11)$$

$$\text{II. } r(t) = h_1(t) * s_1(t) + h_2(t) * s_2(t) + n(t) \quad (12)$$

where $h(t)$, $h_1(t)$ and $h_2(t)$ are the sequences of pulses generated by the sources in Figs. 1 and 2, respectively.

In the one-source case, $h(t)$ is assumed to be a sequence of pulses of varying amplitude as shown in Fig. 5a, but in the two source case, $h_1(t)$ and $h_2(t)$ are assumed to be sequences of pulses of equal amplitude, see Fig. 5b. It is also assumed that the time intervals between successive pulses from each of the sources are longer than $s_1(t)$ and $s_2(t)$.

The application of the pulse shaping technique to the one-source case calls for a filter that estimates $h(t)$ i.e. both the amplitudes a_j and the arrival times θ_j . In the two source case, the amplitude of the pulses are equal and only the arrival times θ_{1j} , θ_{2k} need to be estimated. This can be done by threshold detectors as shown in Fig. 6.

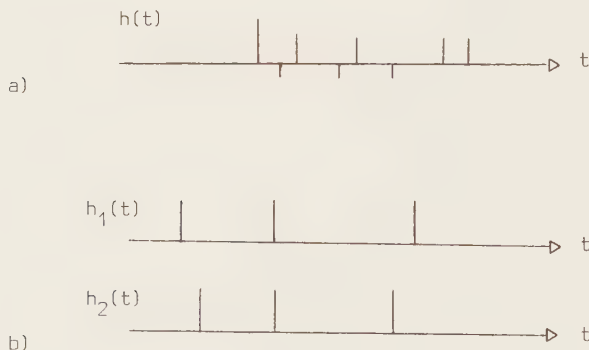


Figure 5. The sequences of pulses from the sources

- a) the one-source case
- b) the two-source case.

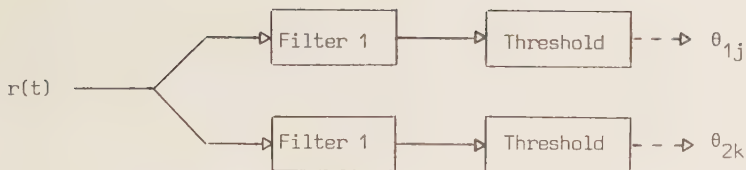


Figure 6. Application of threshold detectors in order to estimate θ_{1j} , θ_{2k} .

Equations (7) and (8) define how the pulse shaping filters are applied in deconvolution. The filters described in the remainder of this paper differ in the specification of the desired output waveform. These specifications are illustrated in Fig. 7. The least squares filter is based on desired output waveforms, Fig. 7a. The envelope-constrained filter and the soft least squares filter are based upon feasible regions, Figs. 7b and 7c. These filters will be applied to both the one-source case and the two-source case (EMG).

In the weighted least squares filter, Fig. 7d, the desired output is unspecified in intervals before and after the main peak. This filter will only be applied to the one-source case (the ultrasonic problem).

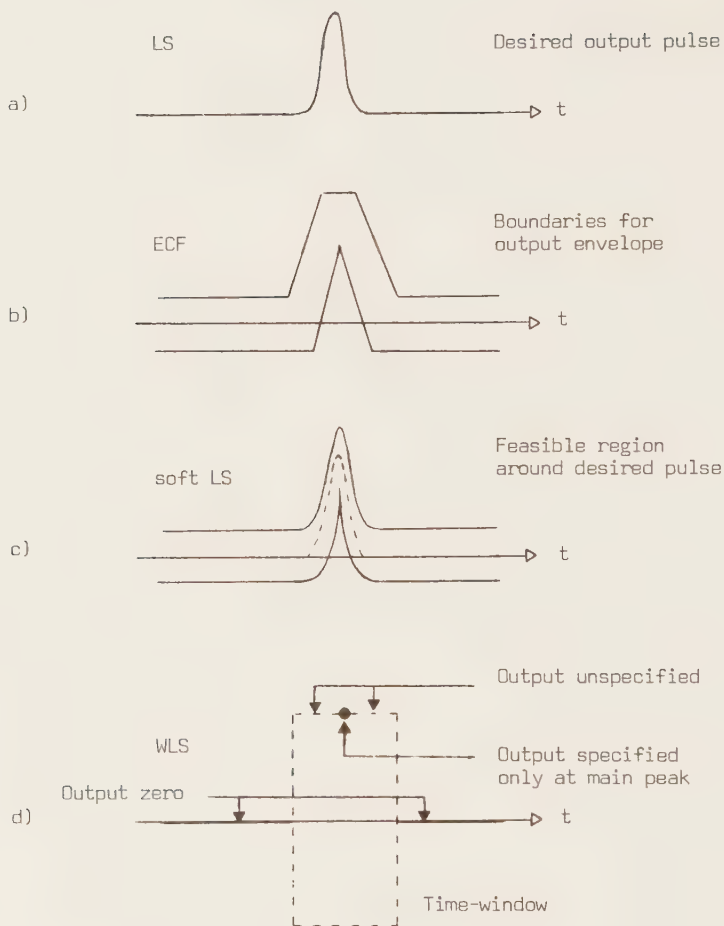


Figure 7. Specifications of the output waveforms for the various approaches.

- a) The least squares filter.
- b) The envelope-constrained filter.
- c) The soft least squares filter.
- d) The weighted least squares filter.

III. THE LEAST SQUARES PULSE SHAPING FILTER

The pulse shaping problem and its applications were defined in Section II. In this section, the least squares pulse shaping filter applied to the resolution of superposed signals will be examined. The least squares cost function was also given in Section II.

The problem is formulated in the time domain and the signals and the filters are assumed to be time-limited. As shown below it is also very important to take the frequency domain properties into account. This section first presents the notations. Then the solution of the filter in the time domain is given and illustrated by examples. The derivation of the filter in the frequency domain using DFT will also be given.

III.1 Notations and definitions

The following vector notations and definitions are used. The filter impulse response $u(t)$ is denoted $\underline{u}$, the input signals $s_i(t)$ are denoted $\underline{s}_i$, $i = 1, 2$, and the desired output waveform $g(t)$ is denoted $\underline{g}$ (Fig. 3). These are defined as column vectors with n , m and $N = n+m-1$ elements, respectively. That is

$$\underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}, \quad \underline{s}_i = \begin{bmatrix} s_1^{(i)} \\ s_2^{(i)} \\ \vdots \\ s_m^{(i)} \end{bmatrix}, \quad \underline{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}, \quad \begin{matrix} N = n+m-1 \\ i = 1, 2 \end{matrix} \quad (13)$$

The noiseless outputs $y_i(t) = s_i(t) * u(t)$ can be written as

$$\underline{y}_i = \underline{S}_i \cdot \underline{u} \quad (14)$$

with the matrices $\underline{S}_i$ defined by

$$\tilde{S}_1 = \begin{bmatrix} s_1^{(i)} & . & . & . & s_1^{(i)} \\ s_2^{(i)} & . & . & . & s_2^{(i)} \\ \vdots & & & & \vdots \\ s_m^{(i)} & . & . & . & s_m^{(i)} \end{bmatrix} \quad (15)$$

The vectors $\underline{y}_i$ are also column vectors of N elements and the dimension of the matrices $\tilde{S}_i$ are $N \times n$.

With the definitions (13), (14) and (15), the least squares cost function (8) can be written

$$J_{LS} = \|\tilde{S}_1 \underline{u} - \underline{g}\|^2 + \|\tilde{S}_2 \underline{u}\|^2 + w_0 \|\underline{u}\|^2 \quad (16)$$

The norm $\|\underline{x}\|^2$ is the Euclidean vector norm $\underline{x}^T \underline{x}$ ($\underline{x}^T$ denotes the transpose of $\underline{x}$).

III.2 Solution

The filter that minimizes the quadratic cost function (16) is found quite simply by differentiating (16) with respect to $\underline{u}$. Then

$$\nabla J_{LS}(\underline{u}) = 2\tilde{S}_1^T (\tilde{S}_1 \underline{u} - \underline{g}) + 2\tilde{S}_2^T \tilde{S}_2 \underline{u} + 2w_0 \underline{u} \quad (17)$$

The minimum is now found from

$$\nabla J_{LS}(\underline{u}) = \underline{0} \quad (18)$$

which gives the set of linear equations

$$\tilde{A} \underline{u} = \tilde{S}_1^T \underline{g} \quad (19)$$

The correlation matrix $\tilde{A}$ is given by

$$\tilde{A} = (\tilde{S}_1^T \tilde{S}_1 + \tilde{S}_2^T \tilde{S}_2 + w_0 \tilde{I}) \quad (20)$$

The matrix $\underline{A}$ is a symmetric Toeplitz matrix and is thus specified by only one row, i.e. only n values. The matrix A is also invertible if $w_0 \neq 0$ and then

$$\underline{u} = \underline{A}^{-1} \underline{S}_1^T \underline{g} \quad (21)$$

Example 1

In the first example simulated signals $s_1(t)$ and $s_2(t)$ are used. They are plotted in Fig. 8 together with their Fourier Transforms. They are generated in the frequency domain by frequency windows with cosine slopes. Then, after taking the inverse DFT, they are multiplied by a time window to get time-limited signals. The filter $u(t) = u_1(t)$ is shown in Fig. 9 where the noiseless outputs are also given. In this example the signals have the main part of their energy at different frequencies (Fig. 8) and the resulting filter gives a large peak for the input signal $s_1(t)$.

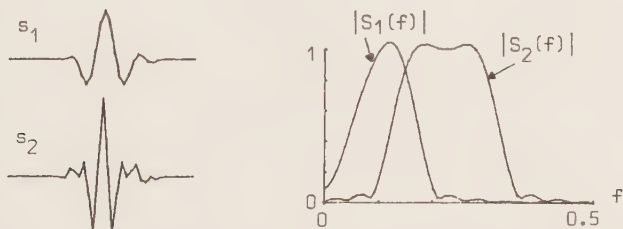


Figure 8. The input waveforms, s_1 and s_2 ($m=40$) and their spectra $S_1(f)$ and $S_2(f)$ computed by an 128 points DFT. $||s_1||^2 = 20$, $||s_2||^2 = 36$.

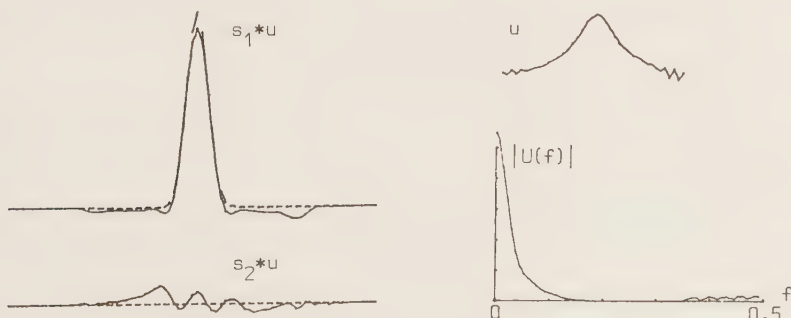


Figure 9. The least squares filter impulse response ($n=40$, $w_0=0.1$) and its spectrum. The desired output waveform $g(t) = e^{-0.1t^2}$ (dashed line).

Example 2

Practical applications of the filters in the resolution of superposed signals can be found in EMG signals (Motor unit potentials). Two estimated waveforms (see [15]), $s_1(t)$ and $s_2(t)$ are shown in Fig. 10. The filters $u(t) = u_1(t)$ and $u(t) = u_2(t)$ are then shown in Figs. 11 and 12, respectively. It is shown that the output peaks for the desired signals ($s_1(t)$ in Fig. 11, $s_2(t)$ in Fig. 12) are about twice as high as the amplitude of other output signals.

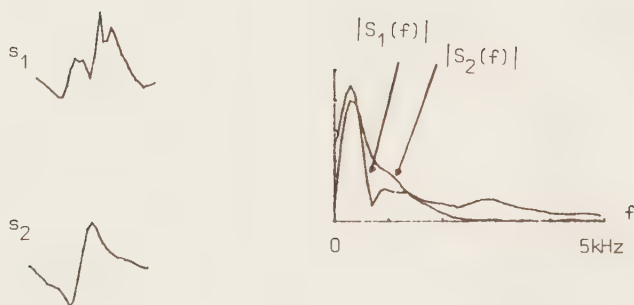


Figure 10. Two estimated waveforms, $s_1(t)$ and $s_2(t)$ and their spectra ($m=32$) (sample rate = 10 kHz).

$$||s_1||^2 = 2.2, ||s_2||^2 = 1.8.$$

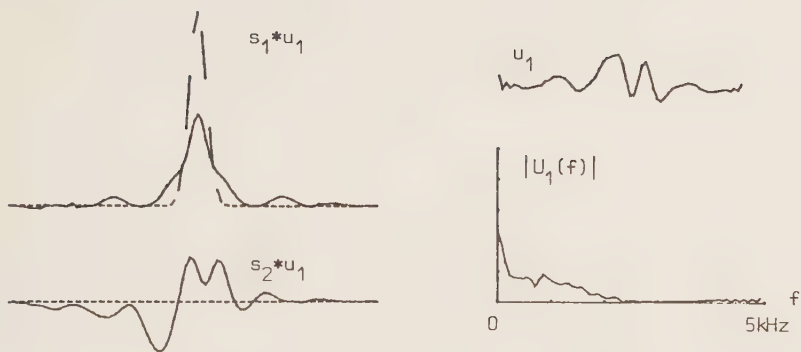


Figure 11. The least squares filter $u_1(t)$ ($w_0 = 0.1$, $n = 64$).

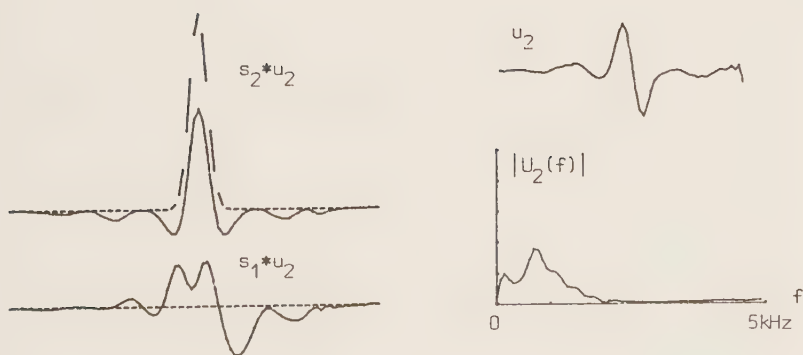


Figure 12. The least squares filter $u_2(t)$ ($w_0 = 0.1$, $n = 64$).

III.3 Derivation of the filter in the frequency domain

The least squares filter can also be computed in the frequency domain [17] using DFT. Then the filter is

$$U(f) = \frac{S_1^*(f)}{|S_1(f)|^2 + |S_2(f)|^2 + R_w(f)} \cdot G(f) \quad (22)$$

where $S_1(f)$, $G(f)$ and $U(f)$ denote the Fourier Transforms of $s_1(t)$, $g(t)$ and $u(t)$, respectively and $R_w(f)$ the input noise power spectrum.

In (22), the filter $u(t)$ is not time-limited (i.e. the convolutions are circular) but the number of points in the DFT are set sufficiently high. An example of this is shown in Section IV.

III.4 The performance of the filters

The value of the minimum cost function does not indicate the performance of the filters accurately. Instead, a measure of the peak amplitude at $t=0$ of $y_1(t)$ in relation to the peak amplitude of $y_2(t)$ is used as a measure of distance. That is

$$D = y_1(t=0) - |y_{2\max}| - |y_{2\min}| \quad (23)$$

This measure of distance is used in Sections IV and V. In the one-source case

$$D = y(t=0) \quad (24)$$

is used, Section VI.

The output noise power is proportional to $||\underline{u}||^2$ (if white input noise) and then

$$\frac{D}{||\underline{u}||} \quad (25)$$

reflects the signal/noise ratio in the output signal. The values of D and $D/||\underline{u}||$ represent the performance of the filters, when different filters are compared with each other.

IV. THE ENVELOPE-CONSTRAINED FILTER

In this section, the application of envelope-constrained filters to the resolution of superposed signals, is studied. The desired output waveforms are now specified by upper and lower boundaries of the envelopes, Fig. 13. Then, these filters are defined as the filters that minimize the output noise power subject to the actual noiseless outputs being inside these boundaries.

In many cases [1] the least squares approach to the pulse shaping filter problem gives large sidelobes in the output waveforms. With the envelope-constrained filters, these sidelobes can be controlled. The feasible regions defined by the boundaries can also lead to the resulting filters having smaller values of $||\underline{u}||$. In other cases, some of the boundary points may be hard to satisfy. However, a routine using the Lagrange multipliers is employed here and such points can be observed as large values of the corresponding multipliers.

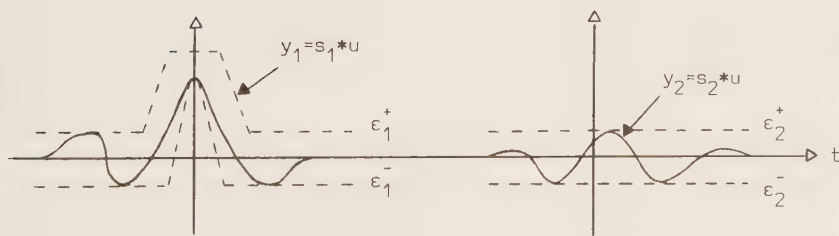


Figure 13. Pulse shape envelopes and noiseless outputs.

IV.1 Definitions

The envelope-constrained filter is defined as the filter that minimizes the output noise power subject to the noiseless outputs being inside the feasible regions defined by the boundaries ϵ_1^+ , ϵ_1^- , ϵ_2^+ and ϵ_2^- . That is

$$\text{minimize } ||\underline{u}||^2$$

subject to

$$\begin{cases} \epsilon_1^-(t) \leq y_1(t) \leq \epsilon_1^+(t) \\ \epsilon_2^-(t) \leq y_2(t) \leq \epsilon_2^+(t) \end{cases} \quad (26)$$

It is convenient to define the desired output waveforms d_i and the error tolerance ϵ_i as

$$\begin{aligned} d_i &= \frac{1}{2} (\epsilon_i^+ + \epsilon_i^-) \\ \epsilon_i &= \frac{1}{2} (\epsilon_i^+ - \epsilon_i^-) \quad i = 1, 2 \end{aligned} \quad (27)$$

Thus the problem is

$$\text{minimize } ||\underline{u}||^2$$

subject to

$$|y_i(t) - d_i(t)| \leq \epsilon_i(t) \quad i = 1, 2 \quad (28)$$

Using the vector notations defined in Section III, the envelope-constrained filter problem now becomes

$$\text{minimize } ||\underline{u}||^2$$

subject to

$$\begin{aligned} \underline{S}_i \underline{u} - \underline{d}_i - \underline{\epsilon}_i &\leq \underline{0} \\ -\underline{S}_i \underline{u} + \underline{d}_i - \underline{\epsilon}_i &\leq \underline{0} \quad i = 1, 2 \end{aligned} \quad (29)$$

where

$$\underline{d}_i = \begin{bmatrix} d_1^{(i)} \\ d_2^{(i)} \\ \vdots \\ d_N^{(i)} \end{bmatrix}; \quad \underline{\varepsilon}_i = \begin{bmatrix} \varepsilon_1^{(i)} \\ \varepsilon_2^{(i)} \\ \vdots \\ \varepsilon_N^{(i)} \end{bmatrix} \quad i = 1, 2 \quad (30)$$

This is a quadratic optimization problem with linear inequality constraints and if a feasible solution exists that satisfies the constraints, then it must be unique [14].

Standard routines for constrained optimization may be used in solving (29). But the constrained problem will be transformed into an unconstrained problem using the Lagrange multiplier. This leads to a suitable filter structure.

IV.2 Solution

The constrained optimization problem (29) is transformed into an unconstrained problem by using the Lagrange multipliers. The Lagrangian [14] for (29) is defined as

$$\begin{aligned} L(\underline{u}, \underline{\lambda}_1^+, \underline{\lambda}_1^-, \underline{\lambda}_2^+, \underline{\lambda}_2^-) = & \underline{u}^T \underline{u} + [\underline{S}_1 \underline{u} - \underline{d}_1 - \underline{\varepsilon}_1]^T \underline{\lambda}_1^+ - [\underline{S}_1 \underline{u} - \underline{d}_1 + \underline{\varepsilon}_1]^T \underline{\lambda}_1^- + \\ & + [\underline{S}_2 \underline{u} - \underline{d}_2 - \underline{\varepsilon}_2]^T \underline{\lambda}_2^+ - [\underline{S}_2 \underline{u} - \underline{d}_2 + \underline{\varepsilon}_2]^T \underline{\lambda}_2^- \end{aligned} \quad (31)$$

where $\underline{\lambda}_i^+, \underline{\lambda}_i^- \geq 0$, $i = 1, 2$, are the Lagrange multiplier vectors in R^N corresponding to upper and lower constraints, respectively. The Lagrangian is convex in $\underline{u}$ and for any fixed $\underline{\lambda}_1^+$ and $\underline{\lambda}_1^-$ it is minimized by

$$\underline{u}_*(\underline{\lambda}_1^+, \underline{\lambda}_1^-, \underline{\lambda}_2^+, \underline{\lambda}_2^-) = -\frac{1}{2} [\underline{S}_1^T (\underline{\lambda}_1^+ - \underline{\lambda}_1^-) + \underline{S}_2^T (\underline{\lambda}_2^+ - \underline{\lambda}_2^-)] \quad (32)$$

Substitution of (32) in (31) yields

$$\begin{aligned}
 \phi(\lambda_1^+, \lambda_1^-, \lambda_2^+, \lambda_2^-) &\triangleq \min_{\underline{u}} L(\underline{u}; \lambda_1^+, \lambda_1^-, \lambda_2^+, \lambda_2^-) = \\
 &= -\frac{1}{4} (\lambda_1^+ - \lambda_1^-)^T \underline{S}_1 \underline{S}_1^T (\lambda_1^+ - \lambda_1^-) - \frac{1}{4} (\lambda_2^+ - \lambda_2^-)^T \underline{S}_2 \underline{S}_2^T (\lambda_2^+ - \lambda_2^-) - \\
 &\quad - \frac{1}{4} (\lambda_1^+ - \lambda_1^-)^T \underline{S}_1 \underline{S}_2^T (\lambda_2^+ - \lambda_2^-) - \frac{1}{4} (\lambda_2^+ - \lambda_2^-)^T \underline{S}_2 \underline{S}_1^T (\lambda_1^+ - \lambda_1^-) - \\
 &\quad - \underline{d}_1^T (\lambda_1^+ - \lambda_1^-) - \underline{d}_2^T (\lambda_2^+ - \lambda_2^-) - \underline{\epsilon}_1^T (\lambda_1^+ + \lambda_1^-) - \underline{\epsilon}_2^T (\lambda_2^+ + \lambda_2^-)
 \end{aligned} \quad (33)$$

The function ϕ in (33) is called the dual function.

Using the Kuhn-Tucker theorem [14, page 216] that states that if constraint qualifications (CQ) are satisfied, necessary and sufficient conditions for $\underline{u}_*$ to solve (29) are that there exist Lagrange multipliers λ_{1*}^+ , λ_{1*}^- , λ_{2*}^+ , $\lambda_{2*}^- \geq 0$ such that

$$\begin{aligned}
 \text{a) } \underline{u}_* &\text{ minimize } L(\underline{u}; \lambda_{1*}^+, \lambda_{1*}^-, \lambda_{2*}^+, \lambda_{2*}^-), \\
 \text{i.e. } \underline{u}_* &= -\frac{1}{2} [\underline{S}_1^T (\lambda_{1*}^+ - \lambda_{1*}^-) + \underline{S}_2^T (\lambda_{2*}^+ - \lambda_{2*}^-)]
 \end{aligned} \quad (34)$$

b) The constraints are satisfied.

c) The Lagrange multiplier vectors can be nonzero only where the constraints are active.

Assuming that $\underline{\epsilon}_i > 0$, $i = 1, 2$, and that a feasible solution exists that satisfies the constraints strictly, then the CQ are satisfied. Using that $\underline{\epsilon}_i > 0$, $i = 1, 2$ and condition c) of the Kuhn-Tucker theorem, the two nonnegative multipliers corresponding to the upper and lower boundaries respectively can be combined into $((\lambda_{i*}^+)^T \lambda_{i*}^- = 0)$

$$\lambda_{i*} \triangleq \lambda_{i*}^+ - \lambda_{i*}^-$$

and

$$|\lambda_{i*}| = \lambda_{i*}^+ + \lambda_{i*}^- \quad i = 1, 2 \quad (35)$$

Substitution of (35) into (33) yields

$$\begin{aligned} \phi(\underline{\lambda}_1, \underline{\lambda}_2) = & -\frac{1}{4} [\underline{\lambda}_1^T \underline{S}_1 \underline{S}_1^T \underline{\lambda}_1 + \underline{\lambda}_2^T \underline{S}_2 \underline{S}_2^T \underline{\lambda}_2 + \underline{\lambda}_1^T \underline{S}_1 \underline{S}_2^T \underline{\lambda}_2 + \underline{\lambda}_2^T \underline{S}_2 \underline{S}_1^T \underline{\lambda}_1] - \\ & - \underline{d}_1^T \underline{\lambda}_1 - \underline{d}_2^T \underline{\lambda}_2 - \underline{\epsilon}_1^T |\underline{\lambda}_1| - \underline{\epsilon}_2^T |\underline{\lambda}_2| \end{aligned} \quad (36)$$

Now, the duality theorem [14, page 224] states that optimal Lagrange multipliers can be obtained by maximizing $\phi(\underline{\lambda}_1, \underline{\lambda}_2)$. Then the optimal solution is

$$\underline{u}_* = -\frac{1}{2} [\underline{S}_1^T \underline{\lambda}_{1*} + \underline{S}_2^T \underline{\lambda}_{2*}] \quad (37)$$

with $\underline{\lambda}_{1*}$ and $\underline{\lambda}_{2*}$ maximizing (36).

The structure of the optimal filter $\underline{u}_*$ implied by (37) is two subfilters in parallel, where $\underline{S}_1^T$ and $\underline{S}_2^T$ correspond to the matched filters for $\underline{s}_1$ and $\underline{s}_2$ respectively. If $n=m$ the subfilter $\underline{S}_1^T \underline{\lambda}$ may be written

$$\begin{aligned} \underline{S}_1^T \underline{\lambda} = \lambda_1 \begin{bmatrix} s_1 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} s_2 \\ s_1 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix} + \dots + \lambda_{n-1} \begin{bmatrix} s_{n-1} \\ s_{n-2} \\ s_{n-3} \\ \vdots \\ \vdots \\ s_1 \\ 0 \end{bmatrix} + \lambda_n \begin{bmatrix} s_n \\ s_{n-1} \\ s_{n-2} \\ \vdots \\ \vdots \\ s_2 \\ s_1 \end{bmatrix} + \\ \lambda_{n+1} \begin{bmatrix} 0 \\ s_n \\ s_{n-1} \\ \vdots \\ \vdots \\ s_3 \\ s_2 \end{bmatrix} + \dots + \lambda_{2n-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ s_n \end{bmatrix} \end{aligned} \quad (38)$$

This is shown in Fig. 14 where T designates delay elements. Note that the components of $\underline{\lambda}_1$ and $\underline{\lambda}_2$ are nonzero only when the constraints are active. In the case where the only active constraint is $\epsilon_1^-(t)|_{t=0}$, the filter $\underline{u}_*$ is simply a filter matched to $\underline{s}_1$ (multiplied by a constant).

Moreover, the magnitude of the multipliers indicate the sensitivity of the cost to changes in the constraints, i.e. a large value of a multiplier magnitude $|\lambda_k^{(i)}|$ indicates a large decrease in the cost $||\underline{u}||^2$ if this error tolerance value is increased, see [14, page 222].

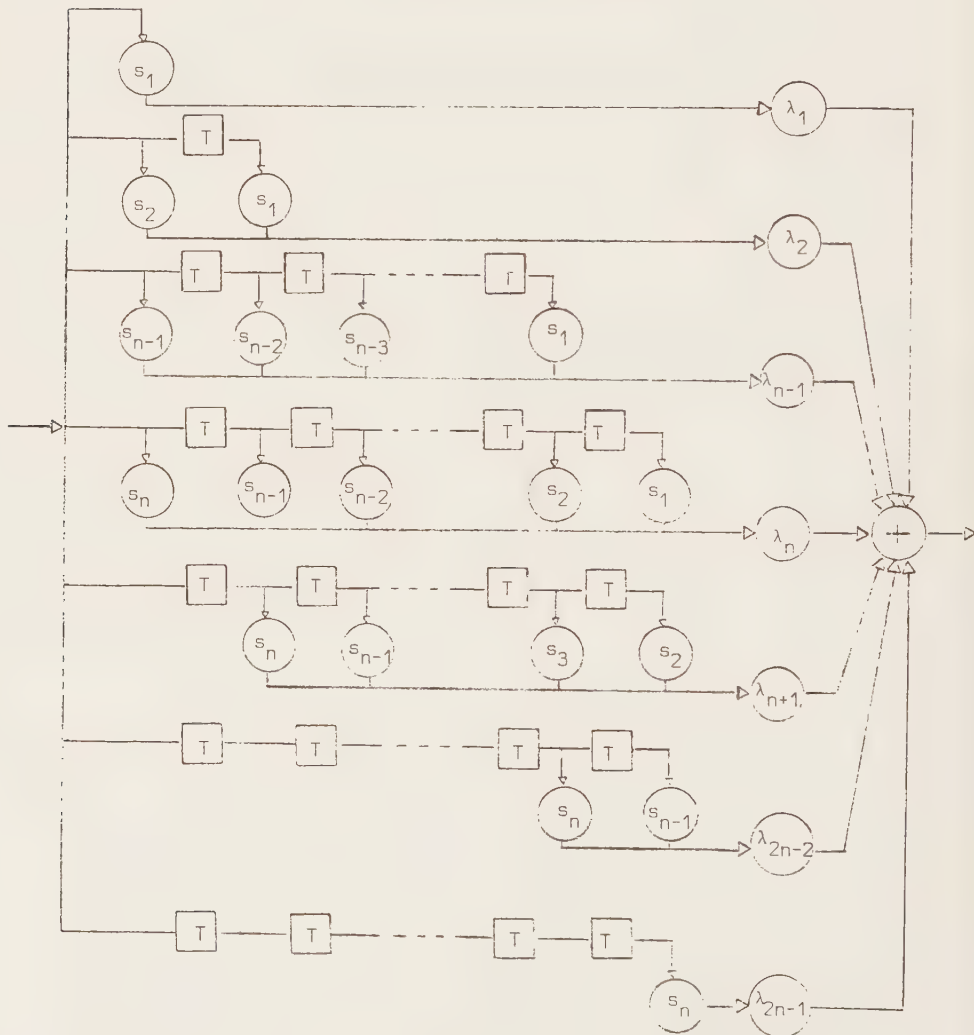


Figure 14. Structure of the subfilter $S_z^T \lambda$.

IV.3 Algorithm

Standard nonconstrained optimization routines can now be used to maximize (36). Unfortunately the dimension of $\underline{s}_i$ and $\underline{u}$ may be large. Here the steepest ascent algorithm with the update formula

$$\underline{\lambda}_i^{k+1} = \underline{\lambda}_i^k + \alpha_i^k \underline{\ell}_i(\underline{\lambda}_1^k, \underline{\lambda}_2^k) \quad i = 1, 2 \quad (39)$$

is used where $\underline{\ell}_i(\underline{\lambda}_1^k, \underline{\lambda}_2^k)$ is an ascent direction and α_i^k is the step size. Using (37), the filter and the outputs in step k are

$$\begin{aligned} \underline{u}^k &= -\frac{1}{2} \underline{S}_1^T \underline{\lambda}_1^k - \frac{1}{2} \underline{S}_2^T \underline{\lambda}_2^k \\ \underline{y}_i^k &= \underline{S}_i \underline{u}^k \end{aligned} \quad i = 1, 2 \quad (40)$$

Combining (39) and (40) the filter update formula is

$$\underline{u}^{k+1} = \underline{u}^k - \frac{1}{2} \alpha_1^k \underline{S}_1^T \underline{\ell}_1(\underline{\lambda}_1^k, \underline{\lambda}_2^k) - \frac{1}{2} \alpha_2^k \underline{S}_2^T \underline{\ell}_2(\underline{\lambda}_1^k, \underline{\lambda}_2^k) \quad (41)$$

In the appendix, an algorithm similar to the algorithm in [1] is derived for the two signal cases. The elements in the ascent direction vector are

$$\left(\underline{\ell}_i(\underline{\lambda}_1^k, \underline{\lambda}_2^k) \right)_j = \begin{cases} y_j^{(i)} - \epsilon_j^{(i)+} & \text{if } \lambda_j^{(i)} > 0 \text{ or if } \lambda_j^{(i)} = 0 \text{ and } y_j^{(i)} > \epsilon_j^{(i)+} \\ y_j^{(i)} - \epsilon_j^{(i)-} & \text{if } \lambda_j^{(i)} < 0 \text{ or if } \lambda_j^{(i)} = 0 \text{ and } y_j^{(i)} < \epsilon_j^{(i)-} \\ 0 & \text{if } \lambda_j^{(i)} = 0 \text{ and } \epsilon_j^{(i)-} \leq y_j^{(i)} \leq \epsilon_j^{(i)+} \end{cases} \quad (42)$$

for $j = 1, 2, \dots, N$ and $i = 1, 2$.

Note that (42) may be interpreted as generalized error vectors. The error is zero if the output is inside the feasible region and the corresponding multiplier is zero. If the output is outside the feasible region, the error is the difference of the output and upper or lower boundaries, respectively, depending on the sign of the multiplier. A block diagram is shown in Fig. 15.

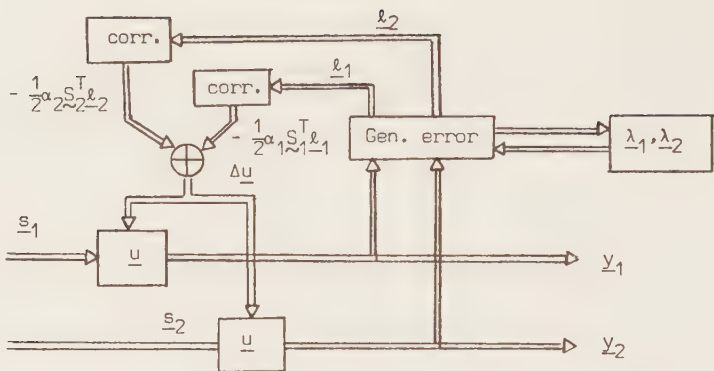


Figure 15. The envelope-constrained filter algorithm. Block diagram.

The solution, (37), is derived assuming the existence of a feasible solution strictly satisfying the boundaries. However, numerical results indicate that the algorithm is robust in the sense that it almost always finds reasonable filters even in the case where no feasible solution satisfying the boundaries exists. This is done by stopping the iterations when no further changes are obtained in $||\underline{u}||^2$ or in the outputs. The robustness can be demonstrated by the case where $\underline{\epsilon}_1$ and $\underline{\epsilon}_2$ are set to $\underline{0}$, forcing the envelope-constrained algorithm to seek the least squares solution.

An example

Before studying performance and alternative formulations an example of the envelope-constrained filter will be given. Figure 16a shows test signals $\underline{s}_1$ and $\underline{s}_2$. These are the same signals as used in the first example in Section III. The resulting filter and corresponding outputs after 1500 iterations are shown in Fig. 16c and d together with the chosen boundaries (dotted lines).

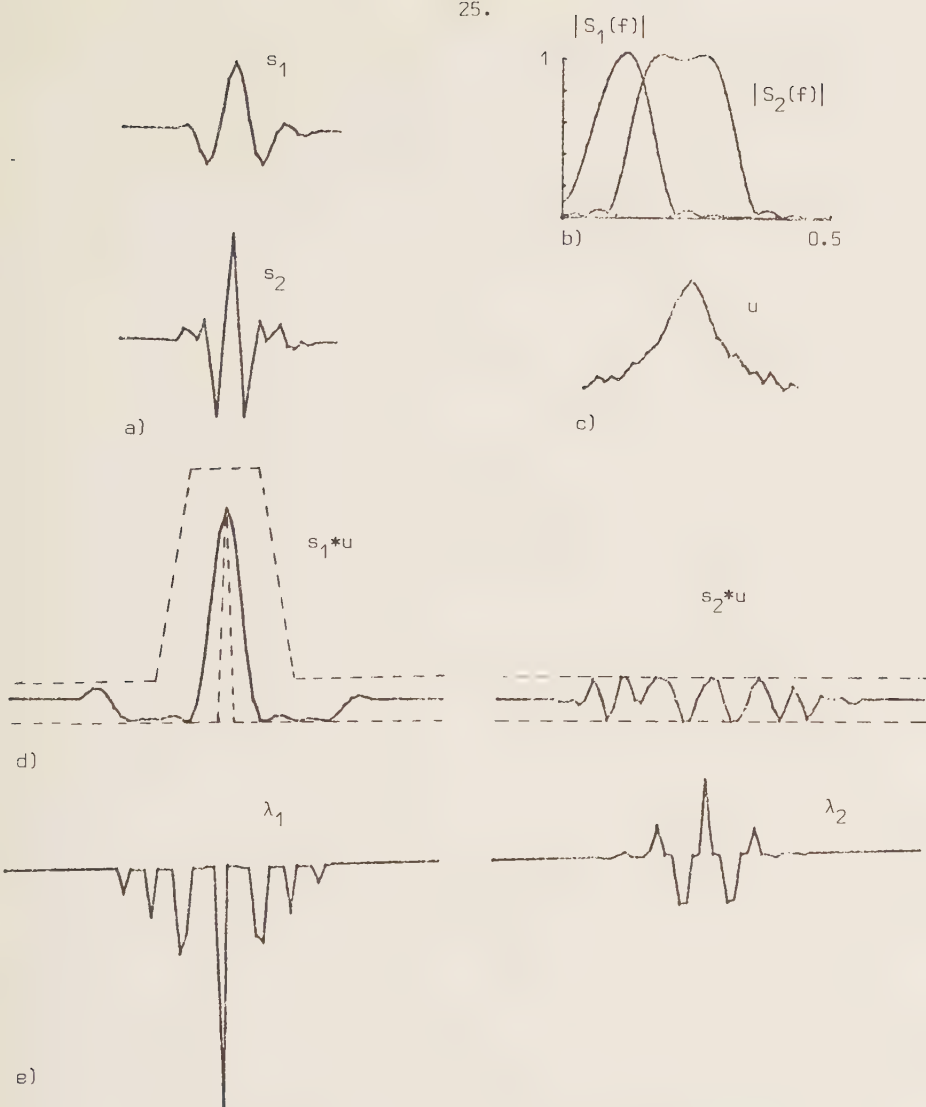


Figure 16. The envelope-constrained filter ($n=m=32$).

- a) waveforms $s_1(t)$ and $s_2(t)$
- b) spectra $S_1(f)$ and $S_2(f)$
- c) the optimum filter $u(t)$
- d) the outputs $y_1 = s_1 * u$ and $y_2 = s_2 * u$
- e) the Lagrange multiplier vectors λ_1 and λ_2 .

IV.4 Alternative formulations

The original formulation is given by (29) where the constraints are $\underline{d}_i \pm \underline{\varepsilon}_i$ ($i = 1, 2$). Then the constraints can be written $\underline{d}_i \pm \xi_i \underline{\varepsilon}_i$ and (29) solved for various values of the scalars ξ_i . This would give a connection between alternative error tolerance $\underline{\varepsilon}_i$ and the corresponding optimal costs $||\underline{u}||^2$. Alternatively, the problem can be written in a minimax form

$$\text{minimize } \alpha ||\underline{u}||^2 + \beta_1 \xi_1^2 + \beta_2 \xi_2^2 \quad (43)$$

$$\text{subject to } |\underline{S}_1 \underline{u} - \underline{d}_1| \leq \xi_1 \underline{\varepsilon}_1$$

$$|\underline{S}_2 \underline{u} - \underline{d}_2| \leq \xi_2 \underline{\varepsilon}_2$$

where ξ_1 and ξ_2 are real variables and α , β_1 and β_2 constants. With $\xi_i^k = \frac{1}{2\beta_i} \underline{\varepsilon}_i^T |\underline{\lambda}_i|$ and $\underline{\varepsilon}_i$ replaced by $\xi_i^k \underline{\varepsilon}_i$ the routine (39-42) will be unchanged.

The unconstrained least squares approach (Section III) to the pulse shaping filter problem is ($w_0=0$)

minimize J with

$$J = ||\underline{S}_1 \underline{u} - \underline{d}_1||^2 + ||\underline{S}_2 \underline{u} - \underline{d}_2||^2 = \sum_{j=1}^N [(y_j^{(1)} - d_j^{(1)})^2 + (y_j^{(2)} - d_j^{(2)})^2] \quad (44)$$

The solution is

$$\underline{u}_* = (\underline{S}_1^T \underline{S}_1 + \underline{S}_2^T \underline{S}_2)^{-1} (\underline{S}_1^T \underline{d}_1 + \underline{S}_2^T \underline{d}_2) \quad (45)$$

if $(\underline{S}_1^T \underline{S}_1 + \underline{S}_2^T \underline{S}_2)$ is invertible.

An adaptive steepest descent algorithm solving (44) is

$$\underline{u}^{k+1} = \underline{u}^k - \alpha' \nabla J(\underline{u}) = \underline{u}^k - \frac{1}{2} \alpha \underline{S}_1^T (\underline{y}_1^k - \underline{d}_1) - \frac{1}{2} \alpha \underline{S}_2^T (\underline{y}_2^k - \underline{d}_2) \quad (46)$$

with $\alpha' = \frac{1}{4} \alpha$ and $\underline{y}_1^k = \underline{S}_1 \underline{u}^k$. In comparison with the envelope-constrained filter the elements in the error vectors in (46) are

$$(\underline{e}_i)_j = y_j^{(i)} - d_j^{(i)} \quad \text{for all } j \quad (47)$$

Setting $\underline{\epsilon}_i = \underline{0}$, $i = 1, 2$ in the routine for the envelope-constrained filter algorithm yields the same error vectors. Thus, the envelope-constrained algorithm seeks the least squares solution in this case even if no solution satisfying the constraints exists.

One combination of least squares cost and the envelope-constrained problem is

$$\begin{aligned} & \text{minimize } ||\underline{S}_1 \underline{u} - \underline{d}_1||^2 \\ & \text{subject to } |\underline{S}_2 \underline{u} - \underline{d}_2| \leq \underline{\epsilon}_2 \end{aligned} \quad (48)$$

This formulation may be useful if $\underline{s}_1$ is the signal of interest and $\underline{s}_2$ is a deterministic disturbance, and then $\underline{d}_2$ is set to zero.

A more general combination of envelope-constraints and least squares costs is the envelope-constrained least squares (ECLS) filter, which is defined as the solution of

$$\begin{aligned} & \text{minimize } \alpha ||\underline{S}_1 \underline{u} - \underline{g}_1||^2 + \beta ||\underline{S}_2 \underline{u} - \underline{g}_2||^2 + \gamma ||\underline{R} \underline{u}|| \\ & \text{subject to } |\underline{S}_1 \underline{u} - \underline{d}_1| \leq \underline{\epsilon}_1 \\ & \quad |\underline{S}_2 \underline{u} - \underline{d}_2| \leq \underline{\epsilon}_2 \end{aligned} \quad (49)$$

where α , β , γ are weights selected by the designer. The desired outputs $\underline{g}_1$ and $\underline{g}_2$ may be equal to (or smoothed versions of) $\underline{d}_1$ and $\underline{d}_2$, respectively.

In (49) $\underline{R}^T \underline{R}$ can be seen as the noise covariance matrix and then $\underline{u}^T \underline{R}^T \underline{R} \underline{u}$ is the output noise power.

The least squares approach may be sensitive to the chosen desired outputs $\underline{g}_1$ and $\underline{g}_2$. Frequency domain considerations are useful in the interpretation of this problem. For example, if $\underline{s}_1$ or $\underline{s}_2$ do not contain certain frequencies, then $\underline{g}_1$ and $\underline{g}_2$ respectively, must be selected to contain none of these frequencies. The term $\underline{u}^T \underline{R}^T \underline{R} \underline{u}$ then makes the solution less sensitive to the chosen desired outputs and even in the noiseless case this term should not be omitted. Assuming white noise we have $\underline{u}^T \underline{R}^T \underline{R} \underline{u} = \text{const } \underline{u}^T \underline{u}$.

The unconstrained (LS) solution of (49) is

$$u_{LS} = (\alpha \tilde{S}_1^T \tilde{S}_1 + \beta \tilde{S}_2^T \tilde{S}_2 + \gamma \tilde{R}^T \tilde{R})^{-1} (\alpha \tilde{S}_1^T \tilde{g}_1 + \beta \tilde{S}_2^T \tilde{g}_2) \quad (50)$$

and the constraint (ECLS) solution is

$$u_{ECLS} = (\alpha \tilde{S}_1^T \tilde{S}_1 + \beta \tilde{S}_2^T \tilde{S}_2 + \gamma \tilde{R}^T \tilde{R})^{-1} \left(\tilde{S}_1^T (\alpha \tilde{g}_1 - \frac{1}{2} \lambda_1) + \tilde{S}_2^T (\beta \tilde{g}_2 - \frac{1}{2} \lambda_2) \right) \quad (51)$$

If no constraints are active then (51) is equal to the least squares solution (50).

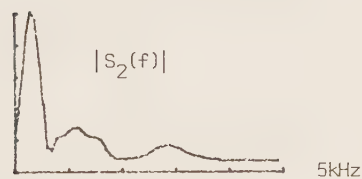
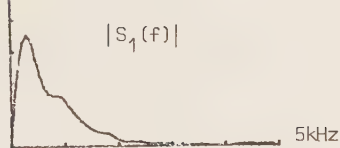
IV.5 Examples

The EMG signals shown in the example in Section III are also used here. They are shown in Fig. 17a. The envelope-constrained filters $u_1(t)$ and $u_2(t)$ are shown in Fig. 17b and c respectively with the noiseless outputs and the boundaries. The lower boundary $\epsilon_1^-(t)$ is taken as 0.9 for $t=0$ and a large negative value for $t \neq 0$. The outputs from filter u_1 are not completely inside the boundaries but even if the algorithm has not found a filter satisfying the boundaries, the resulting filter is quite acceptable.

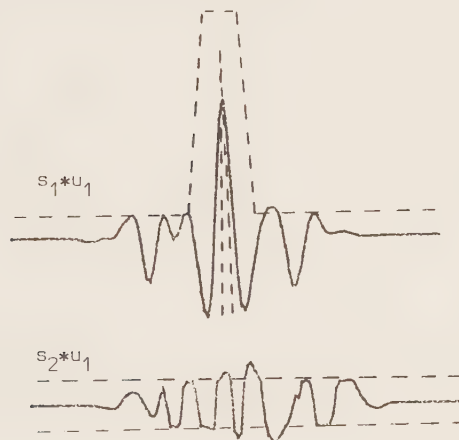
The least squares filters, $U_1(f)$ and $U_2(f)$ computed in the frequency domain by DFT [22] are shown in Fig. 18a (in the frequency domain). The desired output $G(f)$ is a cosine roll-off function shown in Fig. 19. Figure 18b shows the spectra of the envelope-constrained filters in Fig. 17b and c. One difference between the least squares approach and the envelope-constrained approach can be seen for frequencies between 0.4 and 1.1 kHz where the envelope-constrained filter $U_1(f)$ has a higher spectral amplitude than the least squares filter.



a)



b)



c)

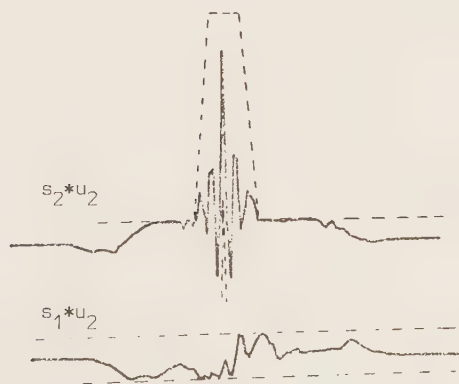


Figure 17. Envelope-constrained filters for EMG signals ($m=n=64$).

a) the signals and their spectra

b) the ECF filter u_{s1} and corresponding outputs

c) the ECF filter u_{s2} and corresponding outputs.

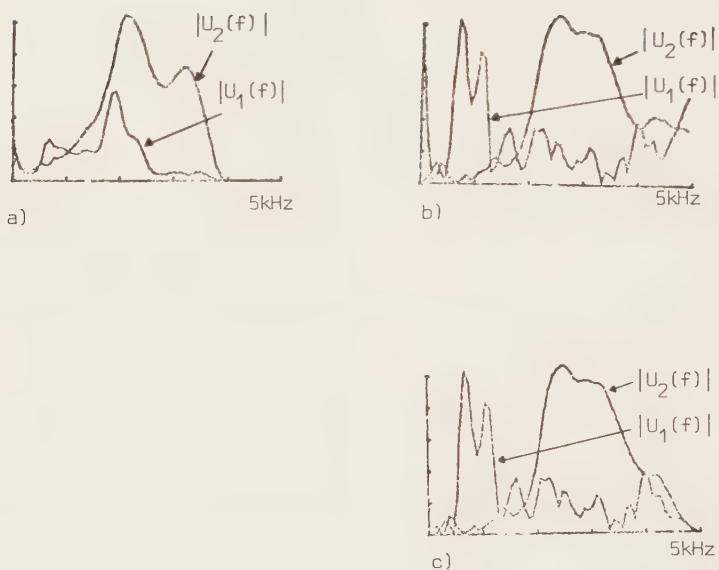


Figure 18. a) The least squares filters in the frequency domain.
 b) The envelope-constrained filters in the frequency domain.
 c) The modified envelope-constrained filters.

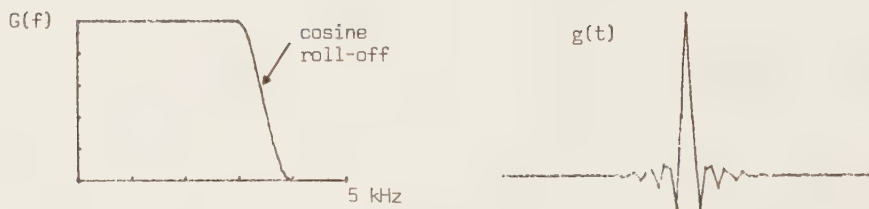


Figure 19. The desired output waveform used in the least squares filters.

Finally the application of the filters to an observed signal containing both $s_1(t)$ and $s_2(t)$ is shown in Fig. 20. Figure 20a shows a time-compressed portion of an EMG recording (the spaces between the individual pulses have been removed). The first two pulses in the observed signal in Fig. 20a are taken to be the signal waveforms $s_1(t)$ and $s_2(t)$ respectively (sample rate equal to 10 kHz).

The applications of the least squares filters and the envelope-constrained filters are shown in Figs. 20b, c, d and e. First the DC component and the highest frequency components are removed (Fig. 18c) in the envelope-constrained filters. Figure 20 shows that the arrival times of $s_1(t)$ and $s_2(t)$ can be estimated by applying thresholds to the filter outputs. Figure 20b and d also shows that the envelope-constrained filter more effectively reduces the output peaks caused by $s_2(t)$ when using the filter $u_1(t)$ compared with the least squares filter.

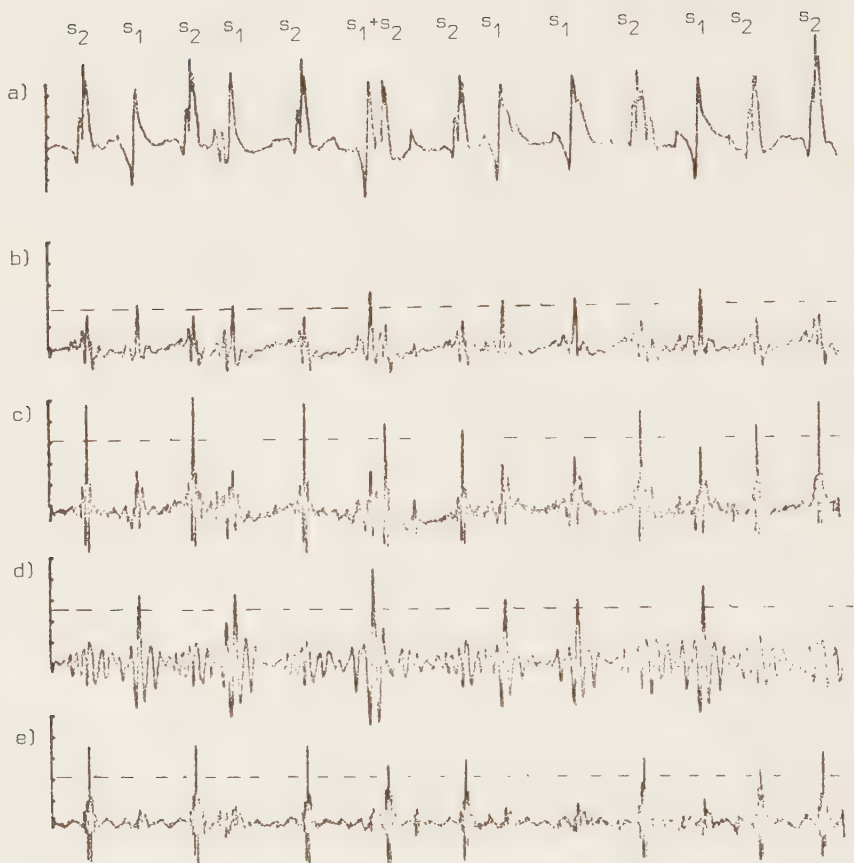


Figure 20. a) A time-compressed portion of an EMG recording consisting of the signals s_1 and s_2 .

b) Output from the LS filter u_1 .

c) Output from the LS filter u_2 .

d) Output from the ECF filter u_1 .

e) Output from the ECF filter u_2 .

V. THE SOFT LEAST SQUARES AND SOFT ENVELOPE-CONSTRAINED FILTERS

This section deals with soft formulations of the least squares and the envelope-constrained filters (see previous sections). In the least squares approach, the desired output waveform is a sharp pulse for one of the inputs. The solutions of the filters may be sensitive to the choice of this desired pulse. In the envelope-constrained filter problem, the desired outputs are replaced by the upper and lower boundaries of the outputs. The least squares approach will now be modified by defining feasible regions around the desired pulse shapes. The errors are set to zero if the outputs lie in these regions. Inserting these regions makes the solutions of the filters less sensitive to the choice of desired output.

The same algorithm as in the least squares problem will also be used for the envelope-constrained approach, except that the feasible regions will be specified by the boundaries given in the previous section. The soft versions of the constrained optimization problem gives an acceptable solution without satisfying the constraints, whether or not there is a true solution that would satisfied the constraints. Further, the soft solution usually reduce the square norm L^2 , which then decreases the output noise power.

V.1 The soft least squares filter

The soft least squares filter is defined from the iterative solution of the least squares filter. The iterative scheme using a steepest descent algorithm to solve the least squares filter (16) is

$$\begin{aligned}\underline{u}^{k+1} &= \underline{u}^k - \frac{1}{2} \alpha \nabla J_{LS}(\underline{u}^k) \\ &= \underline{u}^k - \alpha [\tilde{S}_1^T (\tilde{S}_1 \underline{u}^k - \underline{g}) + \tilde{S}_2^T \tilde{S}_2 \underline{u}^k + w_0 \underline{u}^k] = \\ &= \underline{u}^k - \alpha [\tilde{S}_1^T \underline{e}_1^k + \tilde{S}_2^T \underline{e}_2^k + w_0 \underline{u}^k]\end{aligned}$$

where α is a constant step size and

$$\begin{aligned}\underline{e}_1^k &= \tilde{S}_1 \underline{u}^k - \underline{g} \\ \underline{e}_2^k &= \tilde{S}_2 \underline{u}^k\end{aligned}\tag{52}$$

are the error vectors of the actual outputs and the desired outputs in step k .

Equation (52) is now modified by defining feasible regions around the desired outputs (Fig. 21). The elements of the error vectors $\underline{e}_1^k$ and $\underline{e}_2^k$ are set to zero if the outputs are inside the regions. If the outputs are outside the regions, the errors are defined as the difference between the outputs and the boundaries. Then the elements of the error vector are

$$(\underline{e}_1^k)_j = \begin{cases} (\underline{s}_1 \underline{u}^{k-\underline{g}-\underline{\partial}})_j & \text{if } (\underline{s}_1 \underline{u}^{k-\underline{g}-\underline{\partial}})_j > 0 \\ (\underline{s}_1 \underline{u}^{k-\underline{g}+\underline{\partial}})_j & \text{if } (\underline{s}_1 \underline{u}^{k-\underline{g}+\underline{\partial}})_j < 0 \\ 0 & \text{elsewhere} \end{cases}$$

$j = 1, \dots, N$ (53)

$$(\underline{e}_2^k)_j = \begin{cases} (\underline{s}_2 \underline{u}^{k-\underline{\partial}})_j & \text{if } (\underline{s}_2 \underline{u}^{k-\underline{\partial}})_j > 0 \\ (\underline{s}_2 \underline{u}^{k+\underline{\partial}})_j & \text{if } (\underline{s}_2 \underline{u}^{k+\underline{\partial}})_j < 0 \\ 0 & \text{elsewhere} \end{cases}$$

The vector $\underline{\partial}$ has all its elements equal to ∂ .

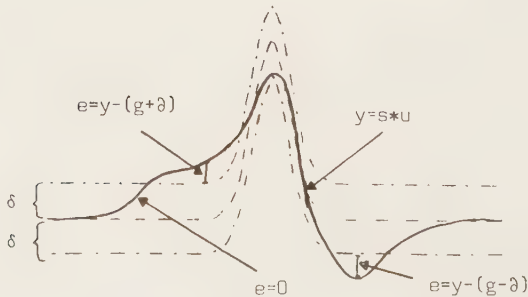


Figure 21. The soft least squares filter.

One effect of inserting the feasible regions is that the square norm $||\underline{u}||$ usually decreases compared to the original LS filter. The algorithm does not try to adjust the outputs exactly to the desired outputs but stops when the boundaries of the regions are reached. Another effect is that the large sidelobes in the outputs, which often occur in the least squares approach, decrease. In the least squares approach, it is known that isolated large errors often have less influence than several small errors. The feasible regions set the small errors to zero and the few large errors can be expected to decrease. Furthermore, the feasible region reduces the sensitivity to the desired output $\underline{g}$. The notation $LS(\partial)$ is used for the soft least squares filter with the regions 2∂ . Inserting the regions corresponds to the problem

$$\begin{aligned}
 \text{minimize } J_{LS(\partial)}(\underline{u}) &= \\
 &= \text{minimize } ||(\underline{S}_1 \underline{u} - \underline{g} - \underline{\partial})_+||^2 + ||(-\underline{S}_1 \underline{u} + \underline{g} - \underline{\partial})_+||^2 + \\
 &\quad + ||(\underline{S}_2 \underline{u} - \underline{d})_+||^2 + ||(-\underline{S}_2 \underline{u} - \underline{d})_+||^2 + \\
 &\quad + w_0 ||\underline{u}||^2
 \end{aligned} \tag{54}$$

where the notation $(\underline{\alpha})_+$ means a vector with the elements

$$\left((\underline{\alpha})_+ \right)_j = \begin{cases} \alpha_j & \text{if } \alpha_j > 0 \\ 0 & \text{if } \alpha_j \leq 0 \end{cases} \tag{55}$$

An example of the filter is given below.

V.2 The soft envelope-constrained filter

The solution of the envelope-constrained filter using the Lagrange multipliers was given in Section IV. This was based on the assumption that there does exist a solution that strictly satisfies the constraints. The constraint optimization problem then become an unconstrained dual optimization problem. A soft solution can be obtained, by adding penalty functions to the cost function. A soft solution tolerates small values outside the feasible regions. The soft envelope-constrained filter, $ECF(w_0)$ is now defined, as the solution of

$$\begin{aligned}
& \text{minimize } J_{\text{ECF}(w_0)}(\underline{u}) = \\
& = \text{minimize } w_0 ||\underline{u}||^2 + ||(\underline{s}_1 \underline{u} - \underline{\varepsilon}_1^+)_+||^2 + ||(-\underline{s}_1 \underline{u} + \underline{\varepsilon}_1^-)_+||^2 + \\
& \quad + ||(\underline{s}_2 \underline{u} - \underline{\varepsilon}_2^+)_+||^2 + ||(-\underline{s}_2 \underline{u} + \underline{\varepsilon}_2^-)_+||^2 \quad (56)
\end{aligned}$$

The parameter w_0 is a weighting factor selected by the designer and $(\underline{a})_+$ is defined by (55). An algorithm analogous to the LS(∂) filter algorithm for the minimization of (56) is

$$\underline{u}^{k+1} = \underline{u}^k - \alpha [\underline{s}_1^T \underline{e}_1^k + \underline{s}_2^T \underline{e}_2^k + w_0 \underline{u}^k]$$

with

$$(\underline{e}_1^k)_j = \begin{cases} (\underline{s}_1 \underline{u}^k - \underline{\varepsilon}_1^+)_j & \text{if } (\underline{s}_1 \underline{u}^k - \underline{\varepsilon}_1^+)_j > 0 \\ (\underline{s}_1 \underline{u}^k - \underline{\varepsilon}_1^-)_j & \text{if } (\underline{s}_1 \underline{u}^k - \underline{\varepsilon}_1^-)_j < 0 \\ 0 & \text{elsewhere} \end{cases} \quad i = 1, 2 \quad j = 1, \dots, N \quad (57)$$

A positive term (the square of the error) is added to the cost function at the points where the corresponding constraints are not satisfied. Now, if w_0 decreases, the influence of the points not satisfying the boundaries increases. As w_0 goes to zero, the solution of (56) goes to the solution of the constrained problem (29). The original problem was to derive filters giving a large peak for one of the input pulses. The choice of the boundaries, is then a somewhat arbitrary and a "soft" solution is adequate.

Equation (56) is very similar to the soft least squares problem (54). The parameter w_0 in $J_{\text{LS}(\partial)}(\underline{u})$ (54) denotes the input noise power. It is consequently to set w_0 in $J_{\text{ECF}(w_0)}$ also equal to the input noise power. The difference between $J_{\text{LS}(\partial)}(\underline{u})$ and $J_{\text{ECF}(w_0)}(\underline{u})$ is how to select the desired output. In LS(∂), the desired output is a specified pulse $g(t)$ for the input $s_1(t)$. The actual outputs differ from $g(t)$ by a value $\pm\partial$ for all t . The parameter ∂ is set by the designer. In ECF(w_0), a feasible region is first selected. This region may be specified by straight lines. Then, w_0 is selected with respect to the trade-off between noise sensitivity and boundary performance. A large value of w_0 means that the resulting filter will have a small value of the square norm $||\underline{u}||$. A small value probably increases $||\underline{u}||$ but also increases the boundary performance.

V.3 Examples

An example of the soft least squares filter and the soft envelope-constrained filter is shown in Figs. 22-26. The selected pulses $\underline{s}_1$ and $\underline{s}_2$ are the same as in Fig. 16, Section IV. The desired output $\underline{g}$ is a gaussian pulse shape normalized to the maximum value equal to 1. The least squares filters are determined for $\vartheta = 0, 0.1$ and 0.2 with $w_0 = 0.1$ (Fig. 22). In Figure 23, the soft envelope-constrained filters for $w_0 = 1, 0.1$ and 0.01 are plotted. The dashed lines are the chosen boundaries. Finally, the envelope-constrained filter determined by the Lagrange multiplier algorithm is plotted in Fig. 24. The resulting distance (23) and the corresponding $||\underline{u}||$ are given in Fig. 26 and plotted in Fig. 25. For small values of w_0 , the $ECF(w_0)$ and the ECF give the same result. With soft solutions, the decreases in $||\underline{u}||$ are greater than the decreases in D (Fig. 25).

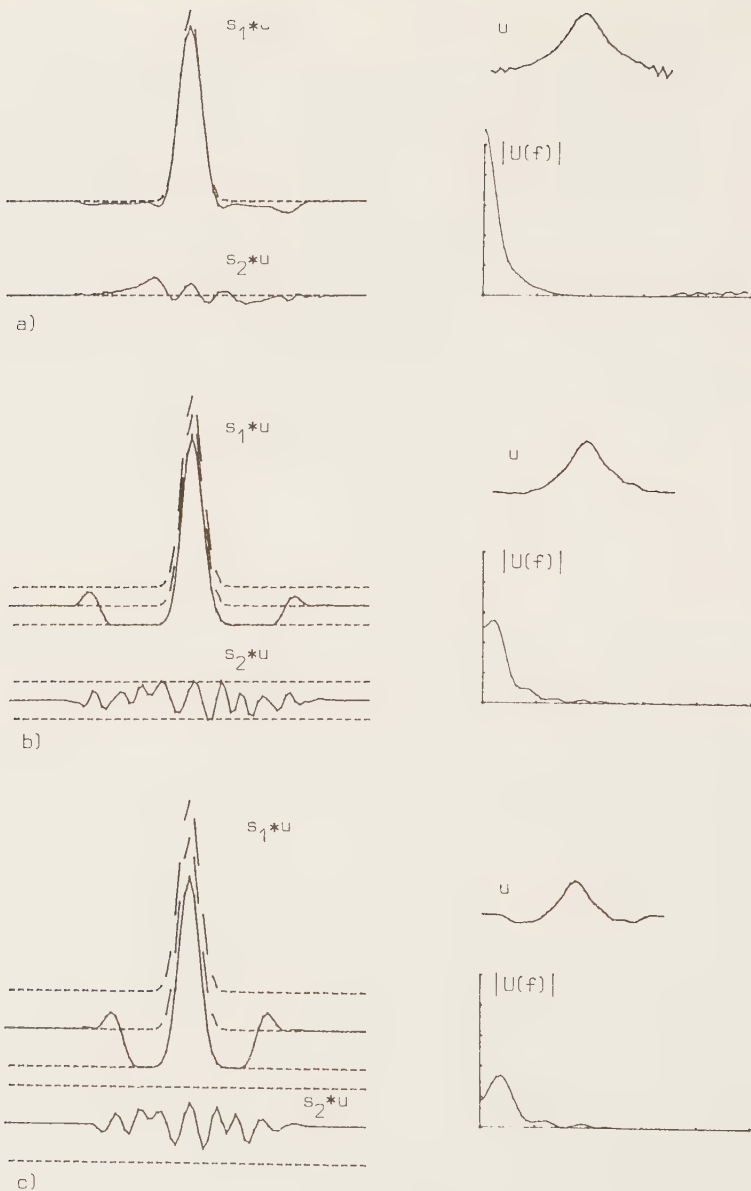


Figure 22. The soft least squares filter ($n=40$). The spectra computed by 128 points DFT. $g(t) = e^{-0.1t^2}$ and $w_0 = 0.1$.
 a) $\delta = 0$, b) $\delta = 0.1$, c) $\delta = 0.2$.

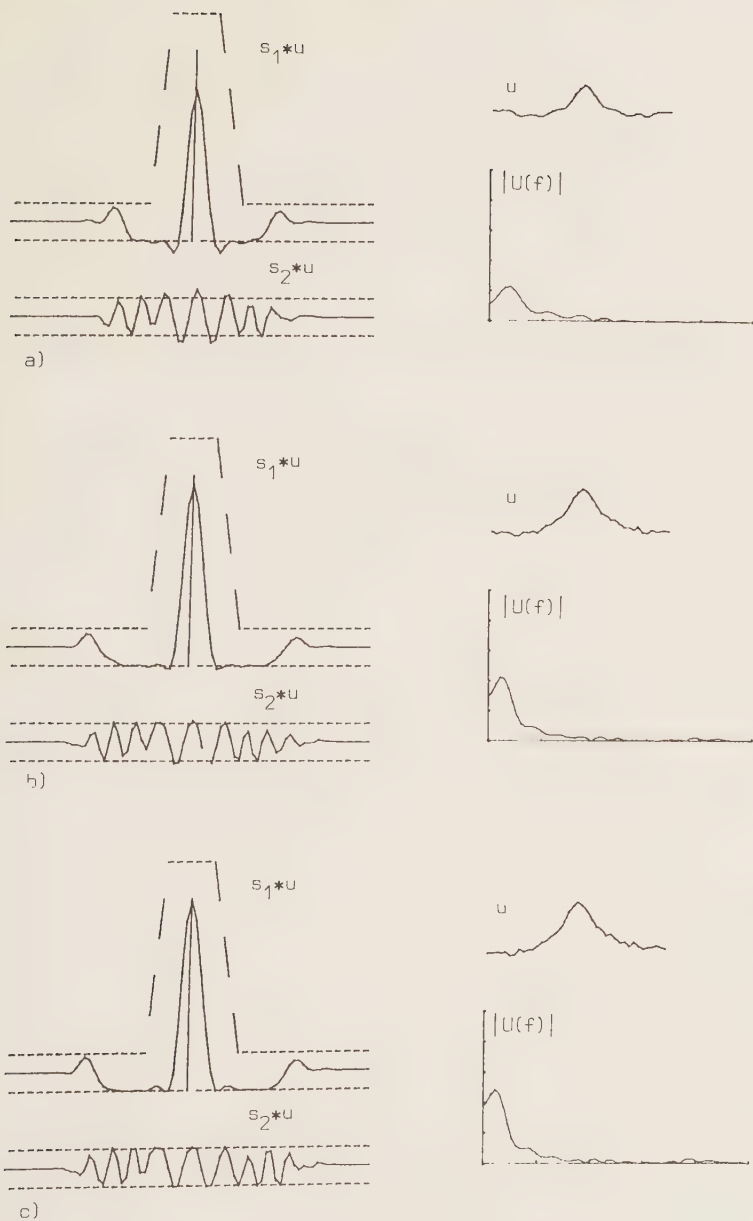


Figure 23. The soft envelope-constrained filter ($n = 40$).

a) $w_0 = 1$, b) $w_0 = 0.1$, c) $w_0 = 0.01$.

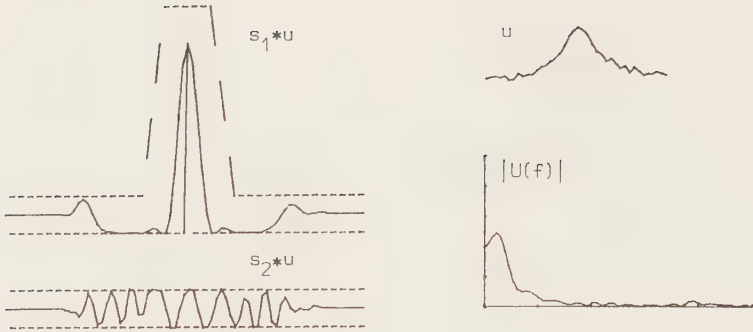


Figure 24. The envelope-constrained filter ($n = 40$) computed by the Lagrange multiplier method.



Figure 25. The performance of the filters.

		$ \underline{u} $	D	$D/ \underline{u} $
Soft least squares filters	$\left\{ \begin{array}{l} \partial=0 \\ \partial=0.1 \\ \partial=0.2 \end{array} \right.$	0.93	0.78	0.84
		0.63	0.69	1.10
		0.41	0.56	1.37
Soft ECF	$\left\{ \begin{array}{l} w_0=1 \\ w_0=0.1 \\ w_0=0.01 \\ w_0=0.001 \end{array} \right.$	0.29	0.42	1.46
		0.48	0.63	1.31
		0.55	0.69	1.26
		0.56	0.70	1.25
ECF using the Lagrangian method		0.56	0.70	1.26

Figure 26. The filter performance.

Further examples are found in [17]. The filters are determined by an iterative steepest descent algorithm. It is shown in [27] that higher rate of convergence is often achieved with the soft formulations compared with the unmodified formulations.

VI. THE WEIGHTED LEAST SQUARES FILTER

Sections III, IV and V apply pulse shaping filters to the problem of resolving superposed signals. This section describes an approach that reduces the duration of the output pulse rather than converts it to some desired shape. It is assumed that there is only one signal source $s(t)$.

The problem of output pulse duration is of great importance for the ultrasonic echo pulse method, since axial resolution is strongly dependent on the duration of the received pulse echoes. A modified least squares filter, the weighted least squares filter, is described. This new filter has in many cases better properties than the original least squares filter, since it embraces a more general class of filters. The least squares filter becomes a special case of the weighted least squares filter.

The weighted least squares filter is based upon time intervals where the output signal may assume arbitrary values. This is specified by a weighting function that is zero in these intervals.

VI.1 Definitions

Recall first the least squares pulse shaping filter. It is defined (for one source) in such a way as to minimize the cost function

$$J_{LS} = ||\underline{\tilde{S}}\underline{u} - \underline{g}||^2 + w_o ||\underline{u}||^2 \quad (58)$$

The cost J_{LS} consists of two components (58). One is proportional to the input noise power. The other is the error between the desired output $g(t)$ and the actual noiseless output $y(t)$. This error can instead be written

$$J_{LS}^S = ||\underline{\tilde{S}}\underline{u} - \underline{g}||^2 = \sum_{t=T_i}^{T_f} [y(t) - g(t)]^2 \quad (59)$$

and is illustrated in Fig. 27. T_i and T_f denote the time interval over which the output is defined (N points).

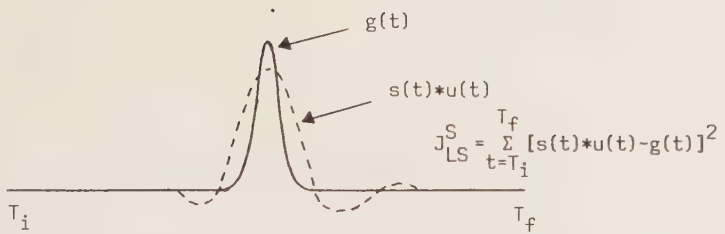


Figure 27. Example of the desired output $g(t)$ and the actual output $y(t) = s(t)*u(t)$.

However, the least squares filter is sensitive to the chosen desired output shape $g(t)$ which in many cases is difficult to select appropriately. This is e.g. the case in the ultrasonic pulse echo method where $s(t)$ contains no low frequency components [20]. However, it is at least possible to decrease the duration of such pulses. This does not refer to the duration of a pulse in a strict sense but rather to the shortest time interval over which the magnitude of the pulse exceeds some threshold value.

One way to decrease the duration of the pulses is to use the weighted least squares filter (WLS). This filter decreases the duration of a pulse instead of converting it to a specified shape. The filter is designed to satisfy the following three conditions:

1. The output must have a high amplitude at one point, say for $t=0$.
2. The amplitude of the output must be low outside some desired resolution interval, say for $|t| > T$.
3. The output noise must be low.

When using the least squares filter conditions 1 and 2 are specified only by the desired shape $g(t)$. However, these conditions also state that outside some achievable resolution interval, the output signal shall be very low while for $T \leq t \leq T$, $t \neq 0$, the output signal is arbitrary. This cannot be taken into account when using the least squares approach. The weighted least squares filter can, however, use this "undefined" interval.

The weighted least squares cost function is defined by

$$\begin{aligned}
 J_{\text{WLS}} &= J_{\text{WLS}}^S + w_0 ||\underline{u}||^2 = \\
 &= \sum_{t=T_i}^{T_f} [w(t) (y(t) - g(t))]^2 + w_0 ||\underline{u}||^2
 \end{aligned} \quad (60)$$

where $w(t)$ is a weighting function and $g(t)$ is assumed to be a unit pulse at $t=0$, i.e. $g(t) = \delta(t)$ ($\underline{g}=\underline{\delta}$) with $\delta(0) = 1$ and $\delta(t \neq 0) = 0$. The weighting function $w(t)$ is selected to be nonzero except for T points each side of $t=0$, as shown in Fig. 28.

The diagonal matrix $\underline{D}$ is defined by

$$(D)_{ii} = w(i) \quad \text{for} \quad i \in T_i, T_f \quad (61)$$

Using the vector notations and (61) the cost function for the weighted least squares filter can be written

$$J_{\text{WLS}} = ||\underline{D}(\underline{S}\underline{u} - \underline{\delta})||^2 + w_0 ||\underline{u}||^2 \quad (62)$$

The filter that minimizes (62) is the solution to the linear equations

$$\underline{A} \underline{u} = \underline{S}^T \underline{D}^T \underline{D} \underline{\delta} \quad (63)$$

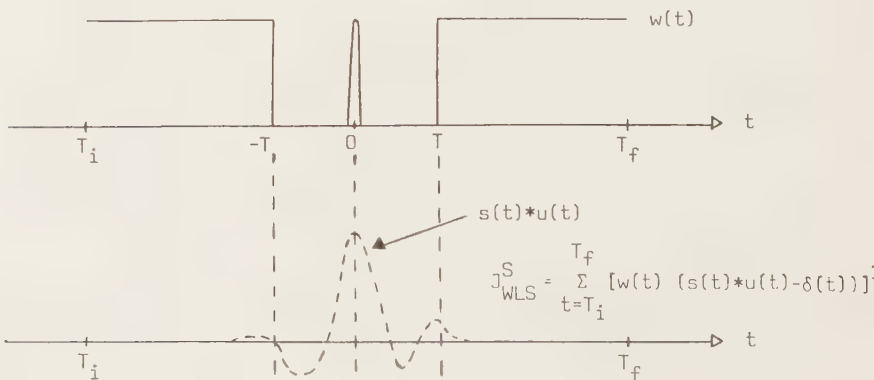


Figure 28. The definition of the weighting function in the WLS filter formulation.

with

$$\underline{A} = (\underline{S}^T \underline{D}^T \underline{D} \underline{S} + w_0 \underline{I}) \quad (64)$$

In the least squares problem (58), the matrix $\underline{A}$ with $\underline{D} = \underline{I}$ is a Toeplitz matrix. The matrix $\underline{A}$ is therefore specified by only one row, i.e. only n values. With $\underline{D} \neq \underline{I}$, the matrix $\underline{A}$ is not Toeplitz but it is still symmetrical and $n \cdot n/2$ values need to be determined. The matrix $\underline{A}$ is invertible if $w_0 \neq 0$ and then

$$\underline{u} = \underline{A}^{-1} \underline{S}^T \underline{D}^T \underline{D} \underline{\delta} \quad (65)$$

The next part of this section deals with some of the properties of the weighted least squares filter.

VI.2 Some properties of the weighted least squares filter

The weighted least squares filter can be seen as a modification of the least squares filter. But it also represents a wider class of filters that includes the least squares pulse shaping filter, the pure inverse filter and the matched filter, as will be seen below. The introduction of the parameter T as described in Fig. 28 has several advantages. The two parameters w_0 and T are "tuned" to give the filter the desired properties:

- (a) high signal/noise ratio,
- (b) good resolution.

Although (a) appears to be specified by w_0 and (b) by T , these parameters do not affect the filter independently of each other.

This "tuning" of w_0 and T is illustrated in a w_0/T diagram, Fig. 29. The pure (time-limited) inverse filter is found by letting both w_0 and T be equal to zero. Then, for large values of T ($w_0 \neq 0$) the output $s(t) * u(t)$ falls entirely within the interval $-T \leq t \leq T$ and only the error at $t=0$ affects the filter. The solution that maximizes the signal/noise ratio at one point is the matched filter, i.e. the time reverse of the signal [13].

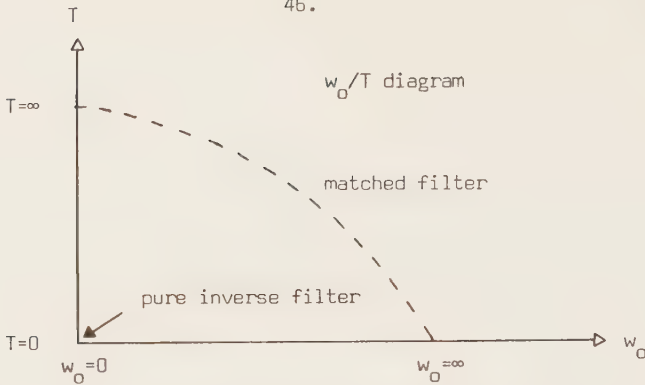


Figure 29. The w_0/T diagram. In the point $T=0$, $w_0=0$ the resulting filter is the pure inverse filter. For large T and/or w_0 , the resulting filter is proportional to the matched filter.

For large values of w_0 , the filter will be approximately proportional to the matched filter. This is because the output amplitude decreases and the error at $t=0$, $s(t)*u(t)-\delta(t)|_{t=0}$, dominates. This can also be seen from the matrix $\tilde{A}$ which will be dominated by $w_0 \tilde{I}$. Then the inverse of $\tilde{A}$ is

$$\tilde{A}^{-1} \approx \frac{1}{w_0} \cdot \tilde{I} \quad (66)$$

Note that

$$\tilde{S}^T \tilde{D}^T \tilde{D} \tilde{\delta} = \tilde{S}^T \tilde{\delta} \quad (67)$$

i.e. one column of the matrix $\tilde{S}^T$ containing the time reverse of $s(t)$.

The w_0/T diagram has the pure inverse filter at the origin. Then, for increasing w_0 and/or T the properties of the filter are changed in such a way that it acquires more and more of the properties of the matched filter. This is shown in an example in the next section. In particular, the plot of the spectra of the filters exhibits these properties (Fig. 32).

In many cases, it is advantageous to know the achieved amplitude at $t=0$ of the noiseless output. This can be done by imposing the constraint $y(0) = 1$. However the weighted least squares filter can incorporate a soft constraint by introducing

$$w'(t) = \begin{cases} 1 & |t| > T \\ a & t = 0, \quad a \geq 1 \\ 0 & |t| \leq T \text{ and } t \neq 0 \end{cases} \quad (68)$$

The contribution of the error at $t=0$ is increased. By using $w(t)$ in Figure 28, (68) can be written

$$w'(t) = w(t) + (a-1) \delta(t) \quad (69)$$

where as before, $\delta(t)$ is the unit pulse. Then the cost function (62) can be written

$$\begin{aligned} J_{WLS} &= \sum_{t=T_i}^{T_f} [w'(t) \cdot (y(t) - \delta(t))]^2 + w_0 ||\underline{u}||^2 = \\ &= \sum_{t=T_i}^{T_f} [(w(t) + (a-1) \delta(t)) (y(t) - \delta(t))]^2 + w_0 ||\underline{u}||^2 = \\ &= \sum_{t=T_i}^{T_f} [w(t) \cdot (y(t) - \delta(t))]^2 + w_0 ||\underline{u}||^2 + (a^2 - 1) (y(0) - 1)^2 \end{aligned} \quad (70)$$

The expression $(a^2 - 1) (y(0) - 1)^2$ is a penalty [14] added to the cost function (62). This soft constraint using of the penalty $(a^2 - 1)$ is in most cases preferable to using $y(0) = 1$ directly in equation (62). The effect of the penalty is an amplification of the filter. An example of this is shown in the next section.

VI.3 Examples

The weighted least squares pulse shaping filter is a suitable approach to the problem of increasing the axial resolution of ultrasonic pulses. The ultrasonic pulse echo method is now widely used in medical diagnosis. However, the axial resolution is strongly dependent on the pulse duration. The duration can be reduced by using the weighted least squares filter and assigning a suitable value to the resolution parameter T . Direct application of inverse filter techniques to ultrasonic signals often leads to unusable filters owing to the bandpass characteristics of the ultrasonic signals.

Figure 30a shows an ultrasonic echo pulse reflected from a plexiglass surface in water. The Fourier transform of the pulse is shown in Fig. 30b. This ultrasonic transmitter/receiver is designed to give a pulse of short duration [20]. The signal is digitized with a sample rate of 10 MHz. The signal is normalized by setting the signal energy to 1, i.e.

$$\sum_t s(t)^2 = 1 \quad (71)$$

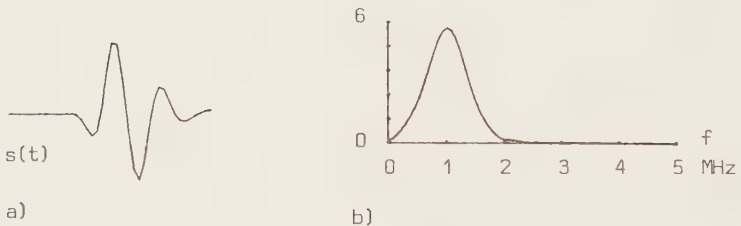


Figure 30. The test signal. The signal is an ultrasonic transmitter/receiver impulse response.

- a) Signal shape (40 points) (sample rate 10 MHz).
- b) The spectrum (computed by a 128 points DFT).

The weighted least squares filter is determined for various values of w_0 and T in equation (62). The impulse responses of the resulting filters are shown in Fig. 31 and the spectra in Fig. 32. The relation of the weighted least squares filters to the inverse filter and the matched filter, respectively, is illustrated best in the frequency domain (Fig. 32). The output from the filters i.e. the convolutions of the filters and the input signal is shown in Fig. 33. The dashed lines enclose the time interval $-T \leq t \leq T$.

The output noise power is proportional to $||\underline{u}||^2$ and

$$\frac{y_{\max}}{||\underline{u}||} \quad (72)$$

reflects the signal/noise ratio in the output signal.

The value of $y_{\max}/||\underline{u}||$ in (72) is shown in Fig. 34 for various values of w_0 and T . Figure 34 shows that $y_{\max}/||\underline{u}||$ is close to 1 for large values of w_0 and/or T , which corresponds to the matched filter. But in these cases there is no decrease in the duration of the output pulse (see Fig. 33).

In this example, the desired output is chosen so that the maximum value of the output signal is equal to one. But the filters are then determined without constraints so that the maximum value of the outputs is normally less than one. The amplitude increases when the approach described in Section VI.3 is used

$$w'(t) = \begin{cases} 1 & |t| > T \\ a & t = 0, \quad a \geq 1 \\ 0 & |t| \leq T \text{ and } t \neq 0 \end{cases} \quad (73)=(68)$$

Figures 35, 36 and 37 show the results for $a = 10$. The shapes of the signals are the same as in Figs. 31, 32 and 33 respectively, but the amplitude has increased.

The examples show how the duration of the output pulse depends on assigning suitable values to w_0 and T . It is not a unique choice of the best filter, which will depend on the actual application. But Fig. 37 demonstrates that a suitable filter is found by using $T=6$ sampling points ($0.6\mu s$) and $w_0 = 0.01$ or 0.1 .

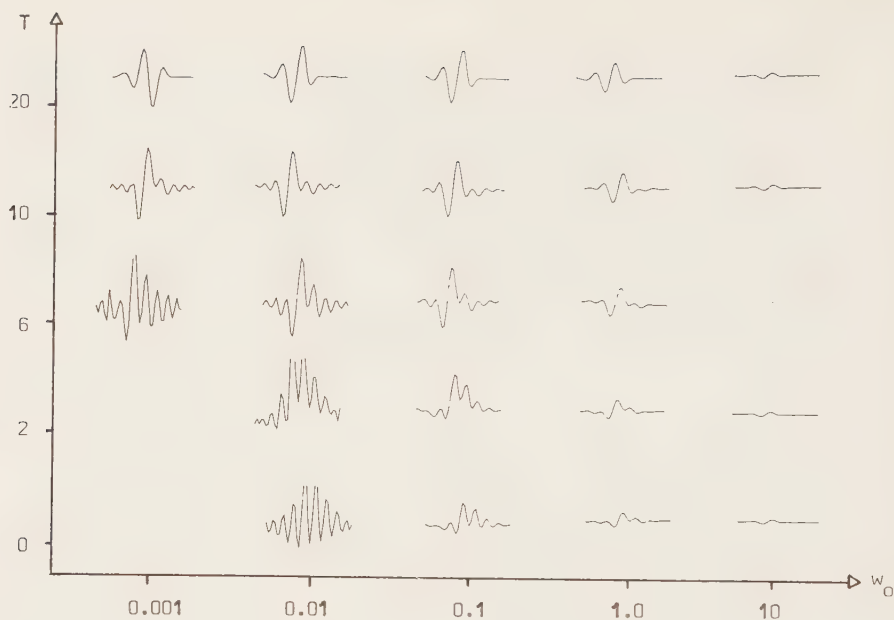


Figure 31. The impulse response of the WLS filters in the w_0/T diagram.

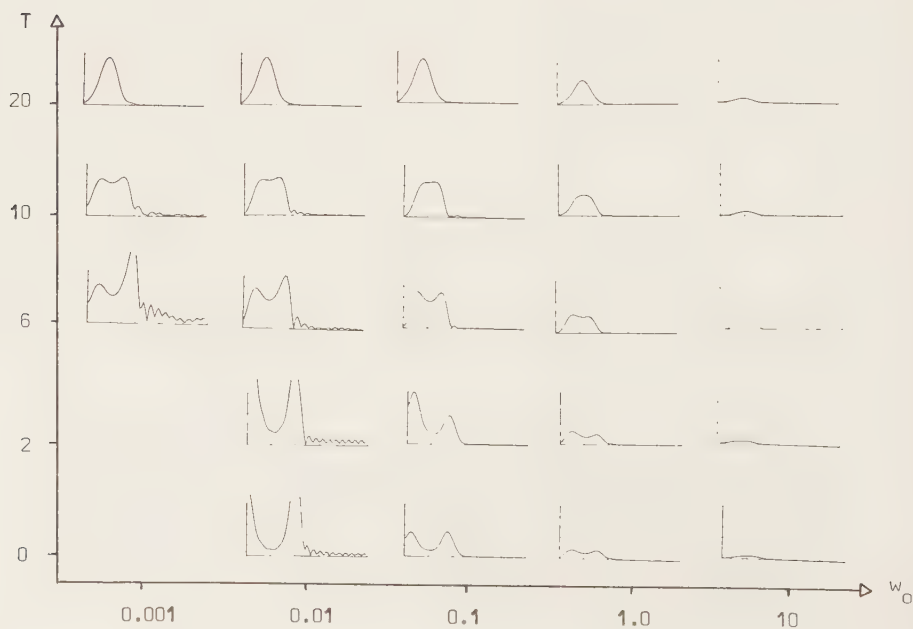


Figure 32. The spectra of the impulse response of the WLS filters in the w_0/T diagram.

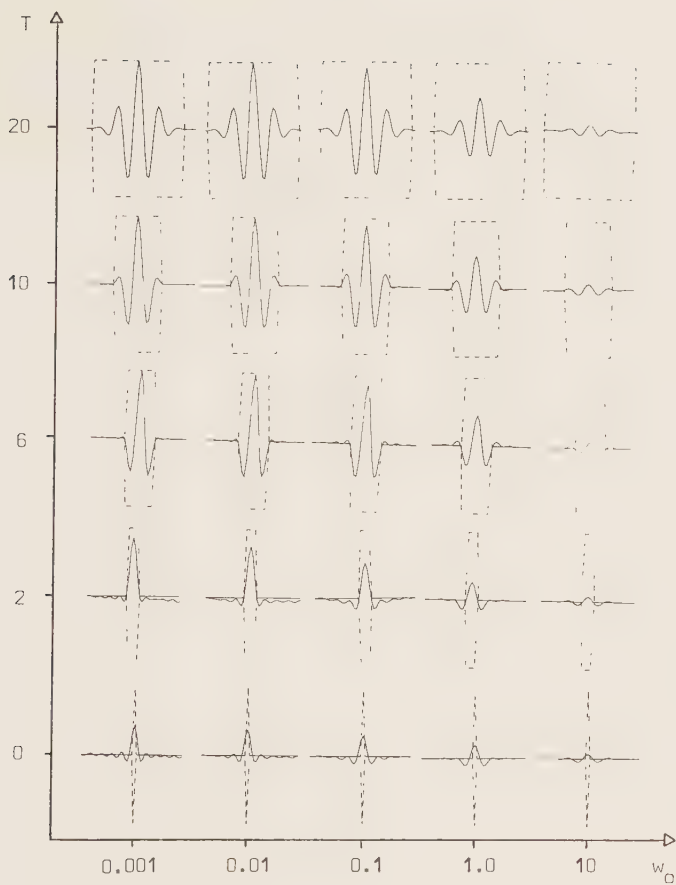


Figure 33. The noiseless outputs from the WLS filters in the w_0/T diagram.

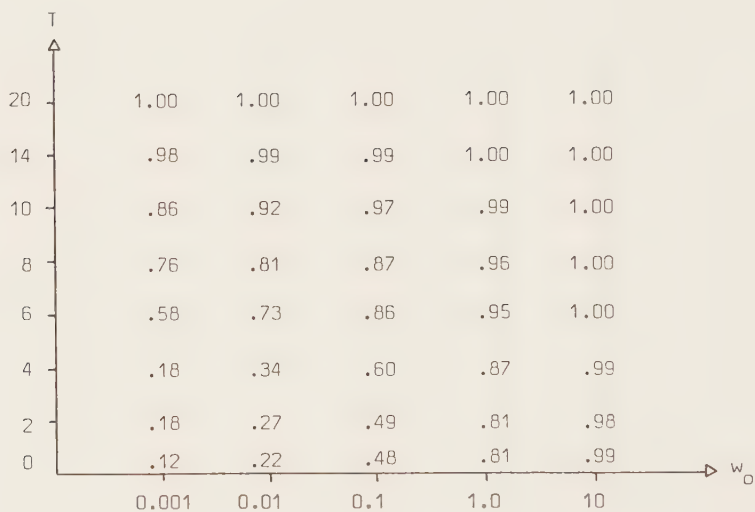


Figure 34. The ratio $y_{\max}/||\underline{u}||$ in the w_0/T diagram.

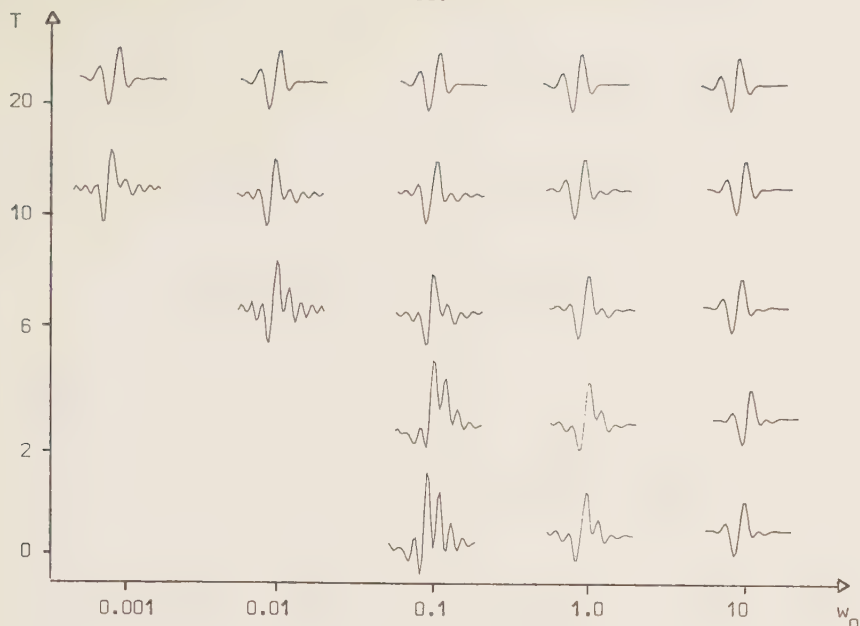


Figure 35. The impulse responses of the WLS filters in the w_0/T diagram with $a = 10$.

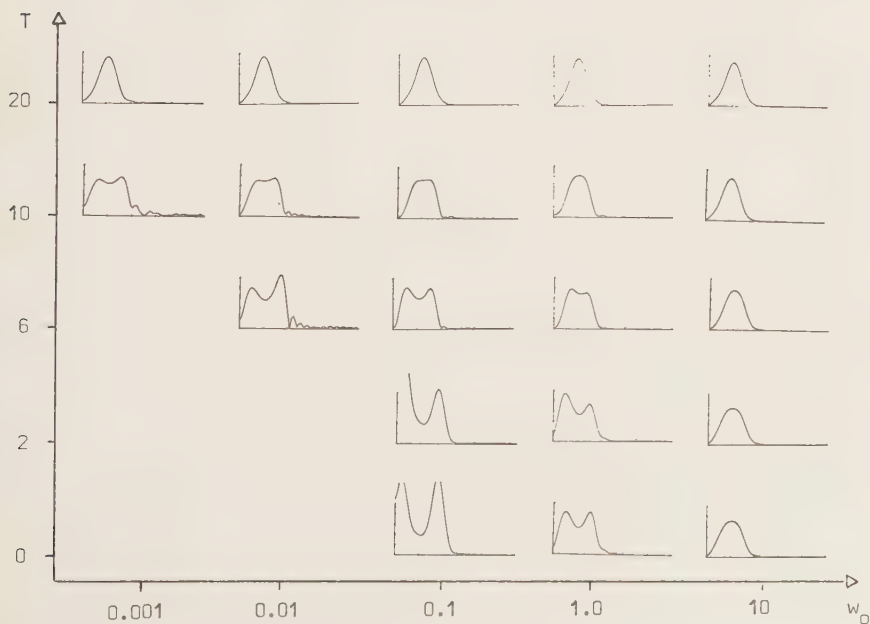


Figure 36. The spectra of the impulse response of the WLS filters in the w_0/T diagram with $a = 10$.

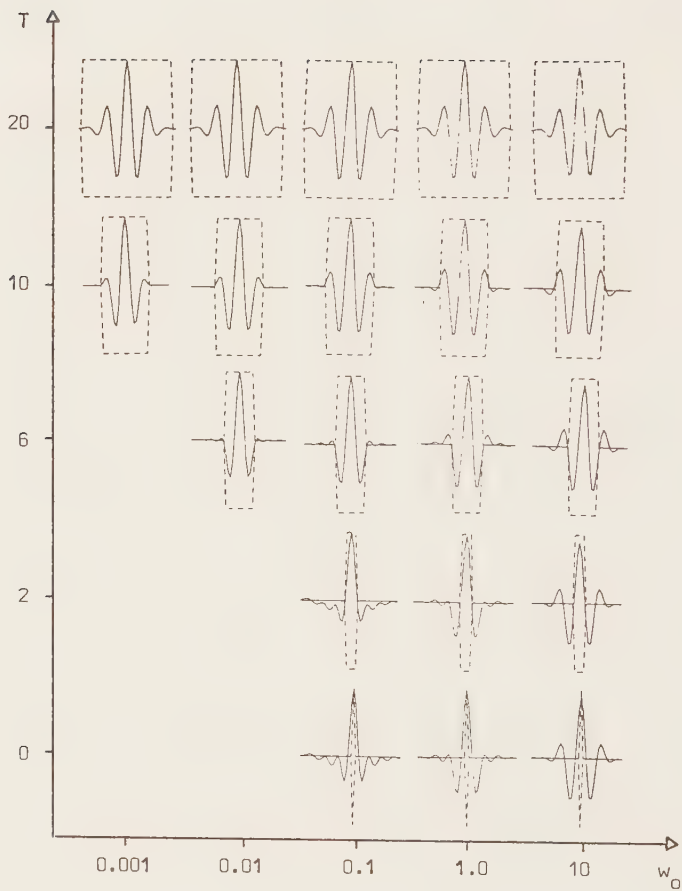


Figure 37. The noiseless outputs from the WLS filters in the w_0/T diagram with $a = 10$.

The w_0/T diagram shows how the duration can be decreased and how this is related to the noise sensitivity of the filters. It will be seen below that it is possible to reduce the number of parameters by using a matched filter in cascade with a digital filter with only a few tap weights.

VI.4 Reduction of the number of parameters

The affinity of the weighted least squares filter to the matched filter can be exploited when designing an application. The number of parameters is then reduced. Indeed it is possible to use only three parameters and still shorten the duration. Consider the configuration shown in Fig. 38. The central branch is simply a matched filter that maximizes the signal to noise ratio at $t=0$ (point B in Fig. 39). For $t > T$ the output must be as low as possible. The upper branch with the delay T_d affects the output in such a way that it cuts it off for $t > T$ (interval C in Fig. 39) by adding a signal with opposite sign to the output from the central branch. Similarly the lower branch with the delay $-T_d$ makes the adjustments to zero for $t < -T$ (interval A in Fig. 39). The value of T_d for which the above holds must be $T_d \approx T$. However T_d must be variable.

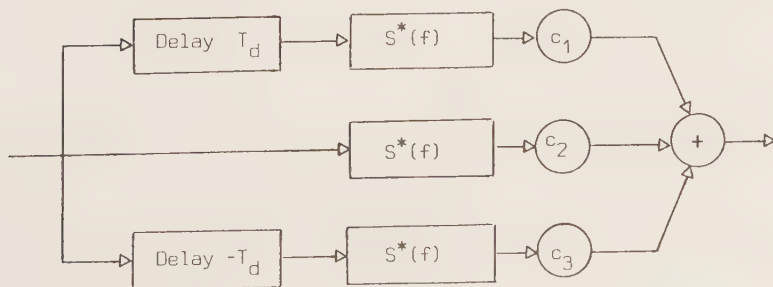


Figure 38. The principle of the use of the matched filter for the pulse shaping problem.

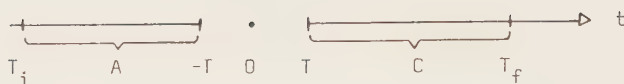


Figure 39. The intervals over which the output is specified.

This configuration is shown in Fig. 40 as a matched filter in cascade with a periodic filter with three parameters. It is shown in [26] that, for problems in synchronous data transmission, a configuration employing a matched filter and a periodic filter can be used to obtain the optimal filter for all practical criteria of goodness.

The configuration in Fig. 40 is obviously not suitable for small values of T and w_0 , i.e. for inverse filters. But it demonstrates that the duration can be reduced with very few parameters provided w_0 and T are not too small. Further tap weights c_j may of course be inserted. For example Fig. 41 shows how tap weights are inserted around $t = -T$ and $t = T$. Similar structure is also described in the design of the envelope-constrained filter in Section IV.

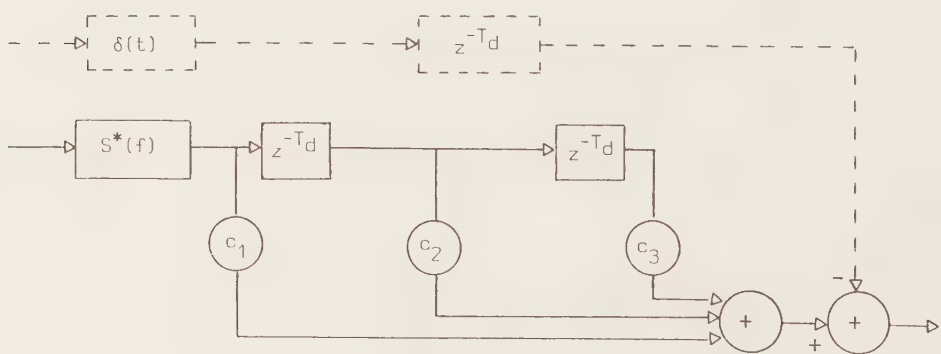


Figure 40. Block diagram of the matched filter application.



Figure 41. An example of time positions of 7 tap weights in the filter following the matched filter.

Let the filter $\underline{u}$ be written as

$$\underline{u} = \underline{Q} \cdot \underline{c} \quad (74)$$

where the columns in $\underline{Q}$ consist of truncated matched filters, i.e. the time reverse of $s(t)$ delayed as defined above. The vector $\underline{c}$ is also a column vector. Inserting (74) in (62) yields

$$J_{\text{WLS}}(\underline{c}) = \|\underline{Q}(\underline{S} \underline{Q} \underline{c} - \underline{\delta})\|^2 + w_0 \|\underline{Q} \underline{c}\|^2 \quad (75)$$

The vector $\underline{c}$ which minimize (75) is

$$\underline{c} = [\underline{Q}^T (\underline{S}^T \underline{Q}^T \underline{Q} \underline{S} + w_0 \underline{I}) \underline{Q}]^{-1} \underline{Q}^T \underline{S}^T \underline{Q}^T \underline{Q} \underline{\delta} \quad (76)$$

and the resulting filter $\underline{u}$

$$\underline{u} = \underline{Q} \underline{c} \quad (77)$$

An example

Choose the same signal as in the previous examples and use $T=6$ and $w_0 = 0.01$. For the configuration in Fig. 40 with three parameters, a value of $T_d=5$ gives the most suitable filter. This is shown in Fig. 42 which also shows the matched filter output. The sidelobes outside $-T \leq t \leq T$ are considerably reduced.

The discussion in this section has not only shown how the number of parameters can be reduced but also how the weighted least squares filter may be more closely related to the matched filter. This structure can of course give very large values of c_j if the number of c_j 's and T_d are not chosen suitably.

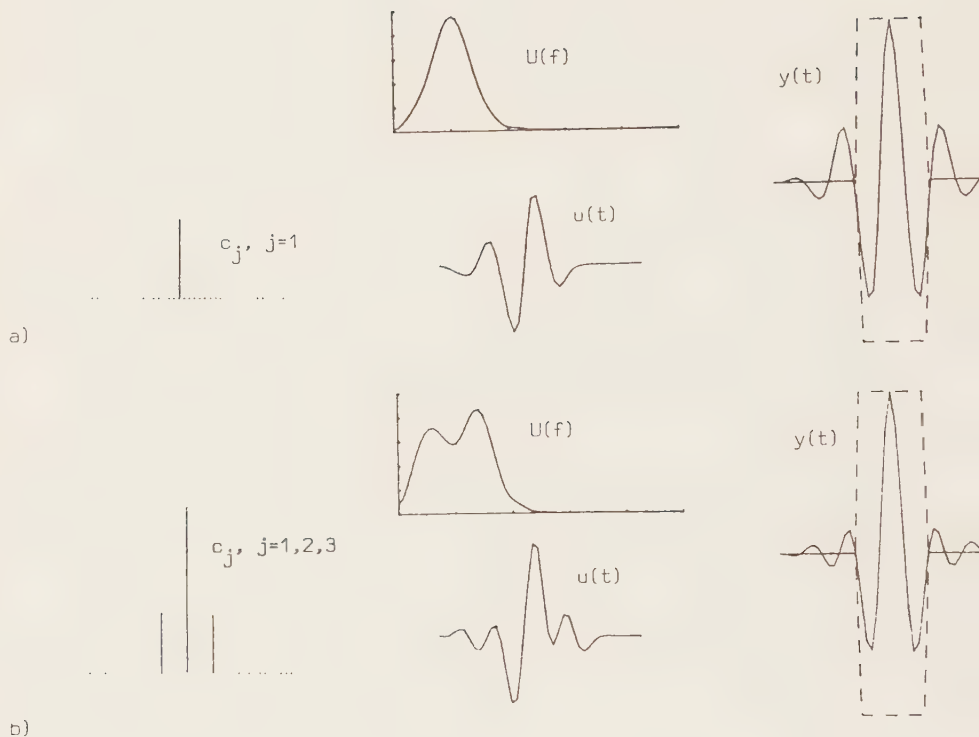


Figure 17. An example of the filter realized a matched filter in cascade with the filter $\underline{c}$.

- a) The matched filter, one tap weight c_1 .
 b) Three tap weights c_j , $j = 1, 2, 3$.

VII. DISCUSSION AND CONCLUSIONS

This paper has dealt with the problem of resolving superposed signals and shortening the duration of pulses. The least squares approach to pulse shaping problems is widely used for this problem. However the least squares approach has many disadvantages such as large sidelobes in the output signal, high sensitivity to noise and often high sensitivity to the desired output waveforms. The modifications presented here have much better properties in many applications.

The envelope-constrained filter problem described in Section IV reduces the sidelobes and also often reduces the noise sensitivity. The method of using the Lagrange multipliers relates the pulse shaping filter to the matched filter. This seems to be very important in designing robust filters.

The "soft" approaches presented in Section V relate the least squares pulse shaping filter to filters specified by boundaries of the output envelopes. With the use of iterative algorithms, this provides the best control of the resulting filters. Of course, these soft approaches are also related to the matched filter in the same way as the envelope-constrained filter although this is not explicitly demonstrated.

Finally the weighted least squares filter described in Section VI offers a very suitable approach for reducing the duration of pulses. Also here, the filter is related to the matched filter.

The pulse shaping problem also involved a comparison of signal error energy and noise power. The actual time interval over which the signals are defined affects the filter. The value of the noise power is represented by the parameter w_0 . But it is not obvious whether the suitable value for w_0 is the actual input noise power or the input noise power multiplied by the length of the current time interval. The approaches described here use w_0 as a parameter to "tune" suitable properties in the resulting filter.

The conclusion that can be drawn from this work is that it is possible to design filters that have much better properties than the least squares filter and that these improved filters can be used to resolve superposed signals.

APPENDIX

An algorithm to solve the dual function

The concave function

$$\begin{aligned} \phi(\lambda_1, \lambda_2) = & -\frac{1}{4} \lambda_1^T S_1 S_1^T \lambda_1 - \frac{1}{4} \lambda_2^T S_2 S_2^T \lambda_2 - \frac{1}{4} \lambda_1^T S_1 S_2^T \lambda_2 - \frac{1}{4} \lambda_2^T S_2 S_1^T \lambda_1 - \\ & - d_1^T \lambda_1 - d_2^T \lambda_2 - \varepsilon_1^T |\lambda_1| - \varepsilon_2^T |\lambda_2| \end{aligned} \quad (A1)$$

is to be maximized with respect to λ_1, λ_2 . The routine described here is similar to the routine in [1].

The directional differential [1] of ϕ with respect to λ_1 and λ_2 at the point λ_1, λ_2 in the direction h_1 and h_2 is defined.

$$\delta_1^+ \phi(\lambda_1, \lambda_2; h_1) \triangleq \lim_{\alpha \rightarrow 0^+} \frac{\phi(\lambda_1 + \alpha h_1, \lambda_2) - \phi(\lambda_1, \lambda_2)}{\alpha} \quad (A2)$$

$$\delta_2^+ \phi(\lambda_1, \lambda_2; h_2) \triangleq \lim_{\alpha \rightarrow 0^+} \frac{\phi(\lambda_1, \lambda_2 + \alpha h_2) - \phi(\lambda_1, \lambda_2)}{\alpha}$$

Necessary and sufficient conditions for the point $\lambda_{1*}, \lambda_{2*}$ to maximize (A1) are

$$\delta_i^+ \phi(\lambda_1, \lambda_2; h_i) \leq 0 \text{ for all } h_i \in R^N \quad i = 1, 2 \quad (A3)$$

The problem (A1) is symmetrical in λ_1 and λ_2 so only the expressions for λ_1 are given.

From the definition (A2)

$$\delta_1^+ |\lambda_k^{(1)}|; h_k^{(1)} = \lim_{\alpha \rightarrow 0^+} \frac{|\lambda_k^{(1)} + \alpha h_k^{(1)}| - |\lambda_k^{(1)}|}{\alpha} = \begin{cases} h_k^{(1)} & \text{if } \lambda_k^{(1)} > 0 \\ -h_k^{(1)} & \text{if } \lambda_k^{(1)} < 0 \\ |h_k^{(1)}| & \text{if } \lambda_k^{(1)} = 0 \end{cases} \quad (A4)$$

Using (A4) gives

$$\begin{aligned}
 \delta_1^+ \phi(\lambda_1, \lambda_2; h_1) &= -\frac{1}{2} \lambda_1^T \tilde{S}_1 \tilde{S}_1^T h_1 - \frac{1}{4} h_1^T \tilde{S}_1 \tilde{S}_2^T \lambda_2 - \frac{1}{4} \lambda_2^T \tilde{S}_2 \tilde{S}_1^T h_1 - \\
 &\quad - d_1^T h_1 - \sum_{k=1}^N \epsilon_k^{(1)} \delta_1^+ |\lambda_k^{(1)}; h_k^{(1)}| \\
 &= -[\frac{1}{2} \lambda_1^T \tilde{S}_1 \tilde{S}_1^T + \frac{1}{2} \lambda_2^T \tilde{S}_2 \tilde{S}_1^T + d_1^T] h_1 - \sum_{k=1}^N \epsilon_k^{(1)} \delta_1^+ |\lambda_k^{(1)}; h_k^{(1)}| = \\
 &= - \sum_{\Lambda_1^+} [(\frac{1}{2} \tilde{S}_1 \tilde{S}_1^T \lambda_1 + \frac{1}{2} \tilde{S}_1 \tilde{S}_2^T \lambda_2)_k + d_k^{(1)} + \epsilon_k^{(1)}] h_k^{(1)} - \\
 &\quad - \sum_{\Lambda_1^-} [(\frac{1}{2} \tilde{S}_1 \tilde{S}_1^T \lambda_1 + \frac{1}{2} \tilde{S}_1 \tilde{S}_2^T \lambda_2)_k + d_k^{(1)} - \epsilon_k^{(1)}] h_k^{(1)} - \\
 &\quad - \sum_{\Lambda_1^0} \{[(\frac{1}{2} \tilde{S}_1 \tilde{S}_1^T \lambda_1 + \frac{1}{2} \tilde{S}_1 \tilde{S}_2^T \lambda_2)_k + d_k^{(1)}] h_k^{(1)} + \epsilon_k^{(1)} |h_k^{(1)}|\} \\
 &\hspace{15em} (A5)
 \end{aligned}$$

where the elements $k = 1, 2, \dots, N$ are partitioned into the disjoint sets

$$\begin{aligned}
 \Lambda_1^+ &\triangleq \{k; \lambda_k^{(1)} > 0\} \\
 \Lambda_1^- &\triangleq \{k; \lambda_k^{(1)} < 0\} \\
 \Lambda_1^0 &\triangleq \{k; \lambda_k^{(1)} = 0\} \quad k = 1, 2, \dots, N
 \end{aligned} \hspace{10em} (A6)$$

Further partitioning of Λ_1^0 may be done if the terms in the sum over Λ_1^0 are rewritten as

$$\begin{aligned}
 &- \{[(\frac{1}{2} \tilde{S}_1 \tilde{S}_1^T \lambda_1 + \frac{1}{2} \tilde{S}_1 \tilde{S}_2^T \lambda_2)_k + d_k^{(1)}] h_k^{(1)} + \epsilon_k^{(1)} |h_k^{(1)}|\} = \\
 &= \begin{cases} -[(\frac{1}{2} \tilde{S}_1 \tilde{S}_1^T \lambda_1 + \frac{1}{2} \tilde{S}_1 \tilde{S}_2^T \lambda_2)_k + d_k^{(1)} + \epsilon_k^{(1)}] h_k^{(1)} & \text{if } h_k^{(1)} \geq 0 \\ -[(\frac{1}{2} \tilde{S}_1 \tilde{S}_1^T \lambda_1 + \frac{1}{2} \tilde{S}_1 \tilde{S}_2^T \lambda_2)_k + d_k^{(1)} - \epsilon_k^{(1)}] h_k^{(1)} & \text{if } h_k^{(1)} \leq 0 \end{cases} \\
 &\hspace{15em} (A7)
 \end{aligned}$$

From (A7) the new disjoint sets are

$$\begin{aligned} Q_1^+ &\triangleq \{k \in \Lambda_1^0; -[z_k^{(1)} + d_k^{(1)} + \epsilon_k^{(1)}] > 0\} \\ Q_1^- &\triangleq \{k \in \Lambda_1^0; -[z_k^{(1)} + d_k^{(1)} - \epsilon_k^{(1)}] < 0\} \\ Q_1^0 &= \Lambda_1^0 - Q_1^+ - Q_1^- \end{aligned} \quad (A8)$$

with

$$z_k^{(1)} = -[\frac{1}{2} \tilde{S}_1 \tilde{S}_1^T \underline{\lambda}_1 + \frac{1}{2} \tilde{S}_1 \tilde{S}_2^T \underline{\lambda}_2]_k \quad (A9)$$

An iterative method for maximizing (A7) with respect to $\underline{\lambda}_1, \underline{\lambda}_2$ is

$$\begin{aligned} \underline{\lambda}_i^{j+1} &= \underline{\lambda}_i^j + \alpha_i \underline{\lambda}_i(\underline{\lambda}_1^j, \underline{\lambda}_2^j) \quad i = 1, 2 \\ j &= 1, 2, 3, \dots \end{aligned} \quad (A10)$$

where α_1, α_2 are step sizes and $\underline{\lambda}_1, \underline{\lambda}_2$ are directions in which

$$\delta_i^+(\underline{\lambda}_1, \underline{\lambda}_2) \geq 0 \quad i = 1, 2 \quad (A11)$$

From (A5) and (A7) such direction $\underline{\lambda}_1$ is

$$\lambda_k^{(1)}(\underline{\lambda}_1, \underline{\lambda}_2) \triangleq \begin{cases} -[z_k^{(1)} + d_k^{(1)} + \epsilon_k^{(1)}] & k \in \Lambda_1^+ \cup Q_1^+ \\ -[z_k^{(1)} + d_k^{(1)} - \epsilon_k^{(1)}] & k \in \Lambda_1^- \cup Q_1^- \\ 0 & k \in Q_1^0 \end{cases} \quad (A12)$$

and the analogous direction $\underline{\lambda}_2$

$$\lambda_k^{(2)} \triangleq \begin{cases} -[z_k^{(2)} + d_k^{(2)} + \epsilon_k^{(2)}] & k \in \Lambda_2^+ \cup Q_2^+ \\ -[z_k^{(2)} + d_k^{(2)} - \epsilon_k^{(2)}] & k \in \Lambda_2^- \cup Q_2^- \\ 0 & k \in Q_2^0 \end{cases} \quad (A13)$$

with

$$z_k^{(2)} = -[\frac{1}{2} \tilde{S}_2 \tilde{S}_1^T \underline{\lambda}_1 + \frac{1}{2} \tilde{S}_2 \tilde{S}_2^T \underline{\lambda}_2]_k \quad (A14)$$

The solution of the optimal filter was

$$\underline{u}^* = -\frac{1}{2} \underline{S}_1^T \underline{\lambda}_1^* - \frac{1}{2} \underline{S}_2^T \underline{\lambda}_2^* \quad (\text{A15})$$

and the corresponding outputs

$$y_1^* = \underline{S}_1 \underline{u}^* \quad (\text{A16})$$

$$y_2^* = \underline{S}_2 \underline{u}^*$$

This gives the filter outputs in step j

$$\begin{aligned} y_1^j &= z_1^j \\ y_2^j &= z_2^j \end{aligned} \quad (\text{A17})$$

With the definition of the boundaries, (29,30) the algorithm may be written

$$\underline{\lambda}_i^{j+1} = \underline{\lambda}_i^j + \alpha_i \underline{e}_i(\underline{\lambda}_1^j, \underline{\lambda}_2^j)$$

with

$$e_k^{(i)} = \begin{cases} y_k^{(i)} - \epsilon_k^{(i)+} & \text{if } \lambda_k^{(i)} > 0, \text{ or if } \lambda_k^{(i)} = 0 \text{ and } y_k^{(i)} > \epsilon_k^{(i)+} \\ y_k^{(i)} - \epsilon_k^{(i)-} & \text{if } \lambda_k^{(i)} < 0, \text{ or if } \lambda_k^{(i)} = 0 \text{ and } y_k^{(i)} < \epsilon_k^{(i)-} \\ 0 & \text{if } \lambda_k^{(i)} = 0 \text{ and } \epsilon_k^{(i)-} \leq y_k^{(i)} \leq \epsilon_k^{(i)+} \end{cases} \quad (\text{A18})$$

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LEAST SQUARES FILTERS USING BANDPASS SUBFILTERS

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LEAST SQUARES FILTERS USING BANDPASS SUBFILTERS

Referat (sammandrag)

ABSTRACT

This paper deals with the problem of designing FIR (finite impulse response) filters. A structure of bandpass subfilters in parallel is used in the filter design. With this structure, the rate of convergence for iterative steepest descent algorithm can be improved. This structure also permits the design of a time-limited filter that concentrates the energy to a selectable frequency band. Simulated results will be presented. The computations are made in the frequency domain which reduce the amount of calculations.

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I. INTRODUCTION

This paper deals with the design of time-discrete filters with finite impulse response (FIR), using a steepest descent algorithm, which is often used in iterative solutions of filters. It is shown how the rate of convergence can be increased by configuring FIR filters as bandpass subfilters in parallel.

Many references [3,4,5] point out that as the length of the FIR filter is increased the eigenvectors of the correlation matrix become increasingly sinusoidal. A numerical example of eigenvectors exhibiting this property is given in [2].

The configuration of the filter as bandpass subfilters in parallel also facilitates the design of essentially bandlimited FIR filters. The computations are made in the frequency domain using a FFT routine. The computation effort can then be reduced by taking advantage of the bandpass characteristic of the subfilters.

The iterative algorithm is suitable for the pulse shaping problem or deconvolution of a received signal. This application is illustrated in Fig. 1. Assuming sampled signals, a cost function is defined by

$$J(u) = E \left\{ \sum_t [s(t) + n(t)] * u(t) - g(t) \right\}^2 \quad (1)$$

where E denotes the expected value and $g(t)$ is a desired output waveform. It is also assumed that the input noise is white and that the noise and the signal are uncorrelated.

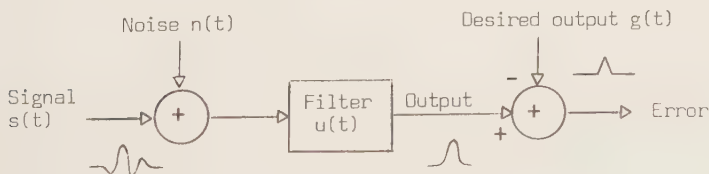


Figure 1. The pulse shaping problem. The filter $u(t)$ increases the resolution of the input by converting it to a sharper pulse at the output.

Then, (1) can be written

$$\begin{aligned}
 J(u) &= \sum_t \left\{ [y(t) - g(t)]^2 + E\{[n(t) * u(t)]^2\} \right\} = \\
 &= \sum_t [y(t) - g(t)]^2 + w_0 \sum_t u(t')^2
 \end{aligned} \quad (2)$$

where w_0 is the expected noise power summed over the interval and $y(t)$ is the noiseless output

$$y(t) = s(t) * u(t) \quad (3)$$

The least squares problem is then defined by

$$\min_u J(u) \quad (4)$$

The block diagram of the application of this filter is shown in Fig. 2.

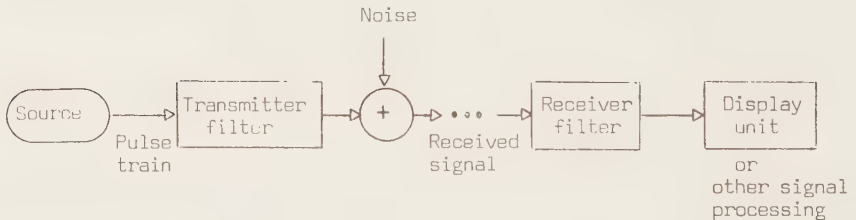


Figure 2. Block diagram of the deconvolution problem.

Section II gives the notations and definitions. Section III gives some properties of nearly diagonal matrices. Examples of bandpass subfilters are given in Section IV and simulation results in Section V.

II. LEAST SQUARES FILTERS

The application of the pulse shaping technique to deconvolution problems yields filters with "inverse" properties relative to the input signals. This means e.g. that if the input signal has a lowpass characteristic then the filter must have a high pass characteristic. Iterative steepest descent algorithms for determining these filters results in low rates of convergence.

The pulse shaping filter problem is used in this paper to demonstrate the higher rate of convergence achieved by configuring the filter as a number of bandpass subfilters in parallel. The pulse shaping problem is described in [1] and [2].

Definitions

The pulse shaping approach is shown in Fig. 1. The input signal $s(t)$ is to be converted by the filter to the desired output waveform $g(t)$. The following vector notations and definitions are used. The input signal is defined by the vector $\underline{s}$. Then, defining the signal matrix $\underline{\tilde{S}}$, the output from the filter $\underline{y}$ can be written

$$\underline{y} = \underline{\tilde{S}} \cdot \underline{u} \quad (5)$$

where

$$\underline{\tilde{S}} = \begin{bmatrix} s_1 & . & . & 0 \\ s_2 & . & . & s_1 \\ \vdots & . & . & \vdots \\ s_m & . & . & s_2 \\ 0 & . & . & s_m \end{bmatrix}; \quad \underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}; \quad \underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \quad (6)$$

The dimension of $\underline{\tilde{S}}$ is $N \times n$, $N = m \cdot n - 1$, with n equal to the length of the filter and m to the length of the input signal. The desired output is denoted by $\underline{g}$ which is a column vector of length N ,

$$\underline{g} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix} \quad (7)$$

With these definitions, the cost function (2) can be written

$$J(\underline{u}) = ||\underline{S} \underline{u} - \underline{g}||^2 + w_0 ||\underline{u}||^2 \quad (8)$$

with $||\underline{x}||^2 = \underline{x}^T \underline{x}$ the Euclidean vector norm ($\underline{x}^T$ denotes the transpose of $\underline{x}$). The solution of

$$\begin{aligned} \min_{\underline{u}} J(\underline{u}) &= \min_{\underline{u}} ||\underline{S} \underline{u} - \underline{g}||^2 + w_0 ||\underline{u}||^2 = \\ &= \min_{\underline{u}} \underline{u}^T (\underline{S}^T \underline{S} + w_0 \underline{I}) \underline{u} - 2 \underline{u}^T \underline{S}^T \underline{g} + \underline{g}^T \underline{g} = \\ &= \min_{\underline{u}} \underline{u}^T \underline{A} \underline{u} - 2 \underline{u}^T \underline{S}^T \underline{g} + \underline{g}^T \underline{g} \end{aligned} \quad (9)$$

is

$$\underline{u}^* = \underline{A}^{-1} \underline{S}^T \underline{g} \quad (10)$$

where the correlation matrix $\underline{A}$ is

$$\underline{A} = (\underline{S}^T \underline{S} + w_0 \underline{I}) \quad (11)$$

In many cases, it is desirable to implement the solution of (9) using an iterative steepest descent method. The steepest descent algorithm with constant step size α is defined by

$$\underline{u}^{k+1} = \underline{u}^k - \frac{1}{2} \alpha \nabla_{\underline{u}} J(\underline{u}) = \underline{u}^k - \alpha \underline{l}(\underline{u}) \quad (12)$$

with

$$\underline{l}(\underline{u}) = \frac{1}{2} \nabla_{\underline{u}} J(\underline{u}) = \underline{S}^T (\underline{S} \underline{u} - \underline{g}) + w_0 \underline{u} \quad (13)$$

(see also Fig. 3).

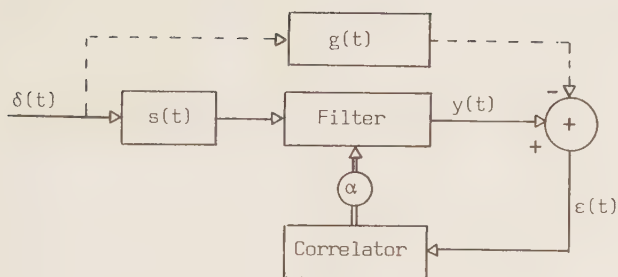


Figure 3. Iterative solution, block diagram.

The rate of convergence of (12) depends on the eigenvalues of the correlation matrix $\underline{A}$ (11). A bound on the rate of convergence can be found from the largest and smallest eigenvalue.

Let $\lambda_{\max}$ and $\lambda_{\min}$ denote the largest and smallest eigenvalues of $\underline{A}$ respectively, and define the ratio

$$\xi = \frac{\lambda_{\max}}{\lambda_{\min}} \quad (14)$$

Then the error of the filter impulse response in step $k+1$,

$$\underline{e}^{k+1} = \underline{u}^{k+1} - \underline{u}^* \quad (15)$$

is bounded by [6]

$$||\underline{e}^{k+1}|| \leq \frac{\xi-1}{\xi+1} \cdot ||\underline{e}^k|| = \beta \cdot ||\underline{e}^k|| \quad (16)$$

with

$$\beta = \frac{\xi-1}{\xi+1} \quad (17)$$

If the ratio of $\lambda_{\max}$ and $\lambda_{\min}$ is large the value of β will be near 1. Equation (16) bounds the rate of convergence. Near the optimal point, it will indicate the actual rate of convergence. However, the most impor-

tant problem in many cases is not this final behaviour but rather the number of iterations needed to get close to the optimal point. This will be discussed in Sections III and IV.

Bandpass subfilters

The rate of convergence can be increased by using the filter structure shown in Fig. 4. The filter is written as a linear combination of N_c subfilters q_j ,

$$\underline{u} = \sum_{j=1}^{N_c} c_j \cdot \frac{1}{\sigma_j} \cdot q_j = \underline{Q} \underline{\Sigma}^{-1} \underline{c} \quad (18)$$

where $\underline{c}$ is a new parameter vector. The subfilters q_j are normalized to fit the relations

$$\sum_{i=1}^n (q_j)_i^2 = 1 \quad \text{for } j = 1, 2, \dots, N_c \quad (19)$$

and σ_j are scaling parameters. The matrix $\underline{Q}$ consists of q_j , $j = 1, 2, \dots, N_c$ in the columns and is a $n \times N_c$ matrix. The scaling matrix $\underline{\Sigma}$ is a diagonal matrix with the diagonal elements σ_j . Inserting (18) in the cost function (8) gives

$$J(\underline{c}) = \|\underline{S} \underline{Q} \underline{\Sigma}^{-1} \underline{c} - \underline{g}\|^2 + w_0 \|\underline{Q} \underline{\Sigma}^{-1} \underline{c}\|^2 \quad (20)$$

Taking the gradient of (20) yields

$$\begin{aligned} \underline{\lambda}(\underline{c}) &= \frac{1}{2} \nabla_{\underline{c}} J(\underline{c}) = \underline{\Sigma}^{-1} \underline{Q}^T \underline{S}^T (\underline{S} \underline{Q} \underline{\Sigma}^{-1} \underline{c} - \underline{g}) + w_0 \underline{\Sigma}^{-1} \underline{Q}^T \underline{Q} \underline{\Sigma}^{-1} \underline{c} = \\ &= \underline{\Sigma}^{-1} \underline{Q}^T \underline{A} \underline{Q} \underline{\Sigma}^{-1} \underline{c} - \underline{\Sigma}^{-1} \underline{Q}^T \underline{S}^T \underline{g} = \\ &= \underline{A}_{\underline{Q}} \underline{c} - \underline{\Sigma}^{-1} \underline{Q}^T \underline{S}^T \underline{g} \end{aligned} \quad (21)$$

with the new correlation matrix

$$\underline{A}_{\underline{Q}} = \underline{\Sigma}^{-1} \underline{Q}^T \underline{A} \underline{Q} \underline{\Sigma}^{-1} = \underline{\Sigma}^{-1} \underline{Q}^T (\underline{S}^T \underline{S} + w_0 \underline{I}) \underline{Q} \underline{\Sigma}^{-1} \quad (22)$$

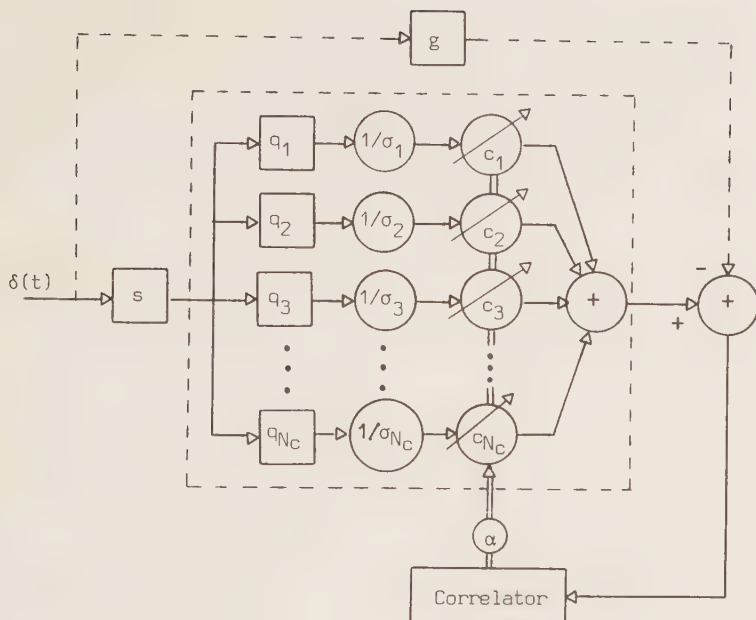


Figure 4. The structure of the iterative solution using a bank of bandpass subfilters in parallel.

The matrix $\tilde{\Sigma}^{-1}$ is chosen such that the diagonal elements of $\tilde{A}_Q$ are equal to 1. This gives

$$(\tilde{\Sigma})_{jj}^2 = \sigma_j^2 = \sum_{\ell} [\sum_k s(k) q_j(\ell-k)]^2 + w_0 = \text{output energy} + w_0 \quad (23)$$

In Section IV various choices of subfilters q_j will be discussed. For a given input signal, setting q_j equal to the eigenvectors of $\tilde{A}_Q$ is the only choice that always makes the correlation matrix $\tilde{A}_Q$ diagonal. Then σ_j are the square root of the eigenvalues, which also satisfies (23). However, the aim is not to find subfilters suitable for only one signal, but for a whole class of input signals.

Several papers [3,4,5] show that the eigenvectors become increasingly sinusoidal as the filter length n increases. This means that the eigenvector subfilters q_j are bandpass signals and the eigenvalues will tend toward the square of the input spectra at the subfilter centre frequencies. In the discussion of alternative choices of Q for a class of inputs, the bandpass subfilter configuration seems to be suitable. In Section III some properties of nearly diagonal matrices will be given.

III. SOME PROPERTIES OF DIAGONALLY DOMINANT MATRICES

The filter is represented as a number of bandpass subfilters in parallel in order to make the correlation matrix $\tilde{A}_0$ to diagonally dominant for various input signals $s(t)$. In this section some properties of diagonally dominant matrices will be illustrated by two examples.

Consider the tridiagonal $n \times n$ matrix $\tilde{A}$,

$$\tilde{A} = \begin{bmatrix} 1 & . & a & & 0 \\ a & . & . & . & \\ & . & . & . & . \\ & & . & . & a \\ 0 & & & a & 1 \end{bmatrix}; \quad a \geq 0 \quad (24)$$

The eigenvalues of the matrix $\tilde{A}$ are bounded by

$$\lambda_{\max} \leq \max_i \left[\sum_{j=1}^n |(\tilde{A})_{ij}| \right] = 1+2a \quad (25)$$

$$\lambda_{\min} \geq \min_i \left[(\tilde{A})_{ii} - \sum_{j=1, j \neq i}^n |(\tilde{A})_{ij}| \right] = 1-2a$$

Now consider two examples with $a=0.5$ and 0.25 , respectively. The bounds given by (25) are

$$\begin{cases} \lambda_{\max} \leq 2 \\ \lambda_{\min} \geq 0 \end{cases} \quad \text{for } a=0.5 \quad (26)$$

and

$$\begin{cases} \lambda_{\max} \leq 1.5 \\ \lambda_{\min} \geq 0.5 \end{cases} \quad \text{for } a=0.25 \quad (27)$$

respectively. The bound of the rate of convergence for the steepest descent algorithm depends on

$$\xi = \frac{\lambda_{\max}}{\lambda_{\min}} \quad (28)$$

$$= (14)$$

and

$$\beta = \frac{(\xi-1)}{(\xi+1)} \quad (29)$$

$$= (17)$$

The values of $\lambda_{\max}$, $\lambda_{\min}$, ξ and β are shown in Table 1 for $n=10$.

First, it is shown that in these cases the bounds (26) and (27) give relatively good approximations of the actual values of $\lambda_{\max}$ and $\lambda_{\min}$, respectively.

A step size

$$\alpha < \frac{2}{\lambda_{\max}} \quad (30)$$

guarantees convergence but prior knowledge of $\lambda_{\max}$ is obviously necessary before the step size α can be chosen. Secondly, Table 1 shows that for $a=0.5$ the value of $\lambda_{\min}$ is near zero ($\beta \approx 1$) and a very slow rate of convergence appears, whereas $a=0.25$ gives $\lambda_{\min} \approx 0.5$ ($\beta \approx 0.5$) and the rate of convergence is greatly increased.

	Bounds on				Actual values for $n=10$			
	$\lambda_{\max}$	$\lambda_{\min}$	ξ	β	$\lambda_{\max}$	$\lambda_{\min}$	ξ	β
$a=0.5$	2.0	0	∞	1.0	1.96	0.04	48	0.96
$a=0.25$	1.5	0.5	3	0.5	1.48	0.52	2.84	0.48
$a=0$	1	1	1	0	1	1	1	0

Table 1. The values and the bounds of the largest and smallest eigenvalues of $\underline{A}$ for $a=0.5$ and $a=0.25$, cf. (25).

Not that for both $a=0.5$ and $a=0.25$ the algorithm can reach a point near the optimal point in a few iterations, but the final adjustment takes many more iterations with $a=0.5$ than with $a=0.25$. This is illustrated in Fig. 5.

The tridiagonal form of the correlation matrix $\underline{A}$ appears if the subfilters are bandpass filters as in Fig. 6. Figure 6 shows subfilters that give values of a in the order of 0.5 and 0.25 respectively, provided the input spectra are constant in the interval around the centre frequencies of the subfilters. Note that Fig. 6 assumes that the time domain signals q_j are not time-limited.

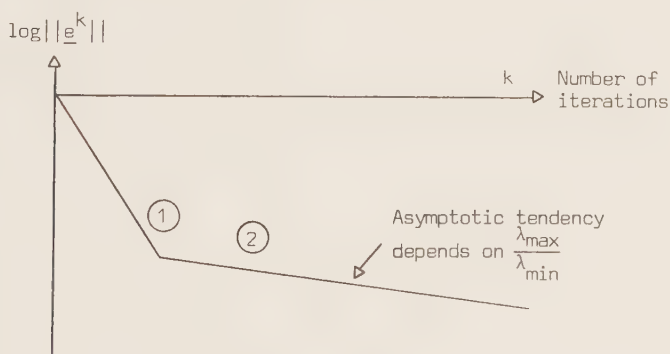


Figure 5. Schematic illustration of the rate of convergence. Slope 1 indicates that relatively few iterations are needed to reach a point near the optimum. Considerably more iterations are required to reach the optimum owing to the asymptotic tendency of slope 2.

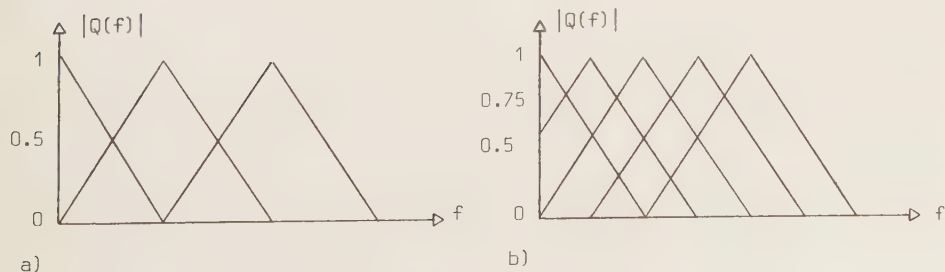


Figure 6. Examples of bandpass signals in the frequency domain

a) gives $a \approx 0.25$,

b) gives $a \approx 0.5$.

In the next section examples of bandpass subfilters will be given. The number of subfilters N_c is assumed to be of the same order as the filter length n . This will give subfilters of the forms shown in Fig. 6b. The forms of the subfilters shown in Fig. 6a can only be implemented if N_c is about $n/2$ (see the next section).

IV. EXAMPLES OF BANDPASS SUBFILTERS

The previous sections have shown how a time-limited filter can be determined by considering it as a bank of bandpass subfilters in parallel. The filter is n points long and the number of bandpass subfilters is N_c , where $N_c \approx n$. First the relation between the time interval and the frequency resolution will be given. Assume that the signal length m and the filter length n are equal. Then the number of nonzero output values is at most $N = m+n-1$. It is assumed that $N = 2 \cdot n$ and that N is a power of 2 so a FFT routine can be used to compute the frequency domain values and to compute the convolution of the signal and the filter. To avoid circular convolution, an N points DFT must be used to convolve an $N/2$ points signal with an $N/2$ points filter.

The definition of the intervals is shown in Fig. 7 for $N=32$. The time interval is $-16 \leq t \leq 15$ (32 points) and the signal and filters are limited to $-8 \leq t \leq 7$. The resolution in the frequency domain with both an $N/2 = 16$ points DFT and an $N=32$ points DFT are shown in Fig. 7.

The first example of subfilters is defined by the $N/2$ points DFT. The filter length is limited in the time domain and this corresponds to multiplying the time domain signals by a rectangular window. A rectangular time window has a spectrum of the form of $\sin(x)/x$, which has large sidelobes. The block diagram for this filter is shown in Fig. 8. The subfilters are $q_j(t) = q_0(t) \cdot \cos(2\pi f_j t)$ where f_j are defined in Fig. 9a. The actual spectra of these time-limited subfilters are shown in Fig. 9b and the first five subfilters in the time domain are shown in Fig. 9c. Observe that the subfilters are zero outside the interval $-8 \leq t \leq 7$. The cosine subfilters are not truncated at the zero crossing points and of course, this increases the effect of the sidelobes at high frequencies of the rectangular time window $q_0(t)$.

Modified DFT subfilters, where the centre frequencies of the sine and cosine subfilters are not the same, are shown in Fig. 10. Here, the centre frequencies are chosen so as to truncate the subfilters at zero crossing points in the time domain. This reduces the sidelobes of the cosine subfilters at high frequencies. With this choice of centre frequencies, only $N/2-1$ subfilters can be determined. The filter length is reduced by one because all subfilters are zero at $t=-N/2$.

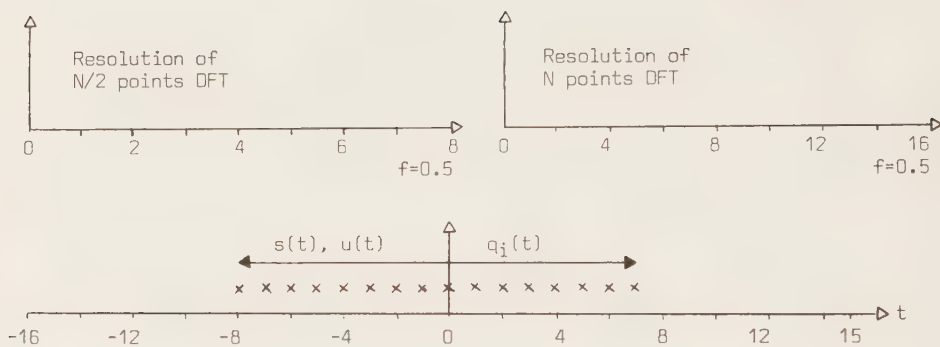


Figure 7. The resolution of the $N/2$ and the N points DFT, $N=32$.

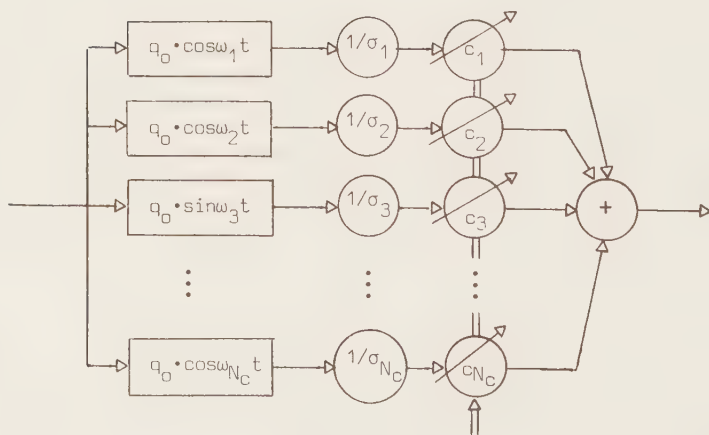


Figure 8. Block diagram of the filter. In the DFT example, q_0 is a rectangular time window and $\omega_1=0$, $\omega_2=\omega_3$, $\omega_4=\omega_5$ and so on.

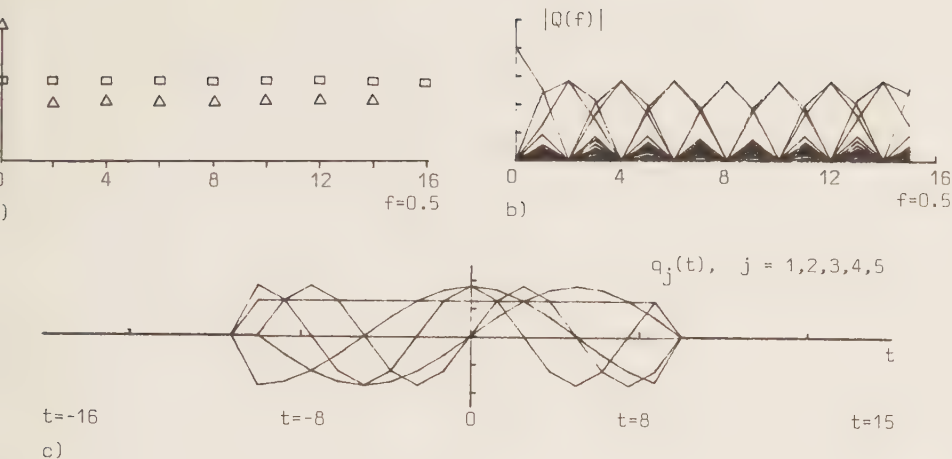


Figure 9. The DFT subfilters, $N=32$,

a) The centre frequencies of the subfilters.

$\square$ = cosine terms, Δ = sine terms.

b) Actual spectra for the time-limited subfilters.

c) The first five subfilters in the time domain.

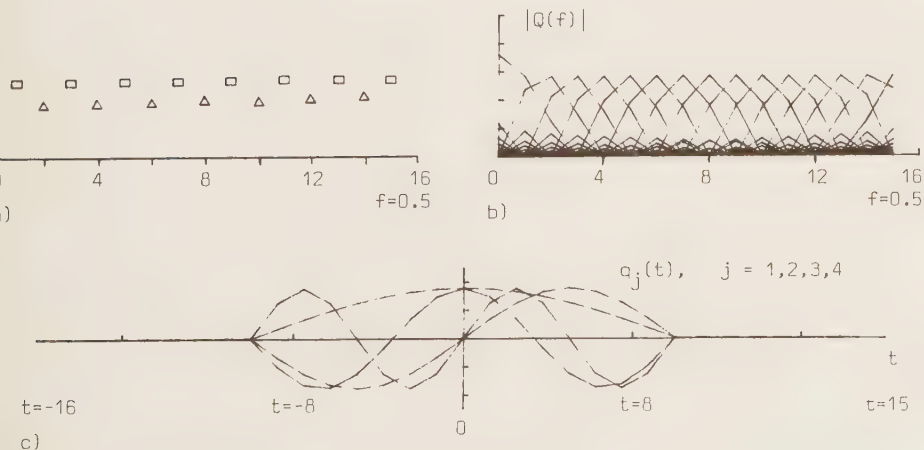


Figure 10. The modified DFT subfilters, $N=32$.

a) The centre frequencies of the subfilters.

$\square$ = cosine terms, Δ = sine terms.

b) Actual spectra for the time-limited subfilters.

c) The first four subfilters in the time domain.

The effect of the different centre frequencies of the DFT and the modified DFT is shown in Fig. 11. Consider a real $Q_0(f)$ of a $\sin(x)/x$ shape. Multiplying $q_0(t)$ by $\cos(2\pi f_j t)$, where f_j denotes the centre frequencies of the DFT and the modified DFT respectively, gives frequency translated $\sin(x)/x$ as shown in Fig. 11a and b. In the DFT case, sidelobes are added with the same sign for frequencies greater than the centre frequencies and opposite sign for frequencies less than the centre frequencies. In the modified DFT case the sidelobes are given opposite sign at frequencies greater than the centre frequency. Multiplying $q_0(t)$ by $\sin(2\pi f_j t)$ gives an odd frequency function and the sign of the added sidelobes is the opposite of that given by cosine multiplication. Note that since the signals are sampled the periodicity around $f=0.5$ (sample rate = 1) gives the opposite effect for high frequencies. However it is assumed that the input signal spectrum is small for frequencies near $f=0.5$.

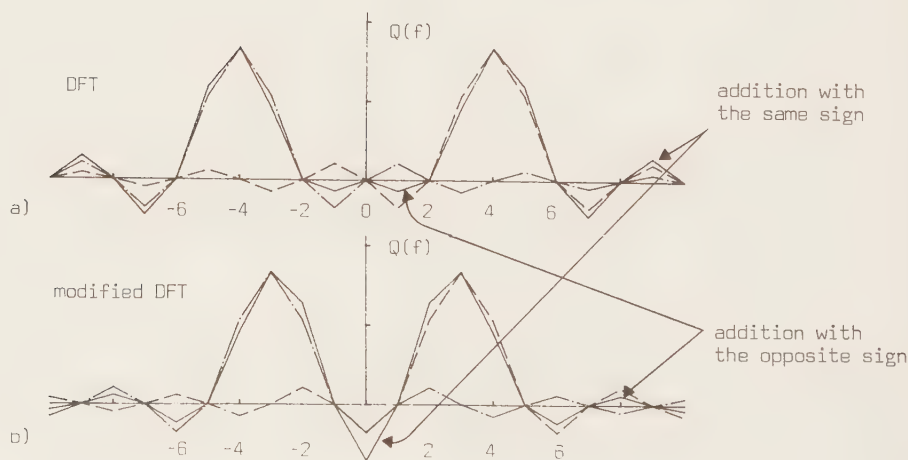


Figure 11. The addition of sidelobes of the cosine terms (real $Q_0(f)$).

a) DFT: Subfilter centre frequency $f_j = 4/N$.

b) Modified DFT: Subfilter centre $f_j = 3/N$.

The large sidelobes of the $\sin(x)/x$ in the DFT and the modified DFT subfilters affect the correlation matrix $\hat{A}_Q$ so that the subdiagonals are not small. In the third example, $q_0(t)$ is chosen to be a half period of a cosine shape (denoted half circle cosine (HCC)). In this example, this is achieved by letting each of the subfilters be the sum of signals with different centre frequencies $(\cos x + \cos y) = 2 \cos(x-y)/2 \cos(x+y)/2$. Figure 12 shows this third example. The spectra of the impulse responses of these filters are much more concentrated around the centre frequencies. In this case $N/2-2$ subfilters may occur, see Fig. 12a. This choice of subfilters does not give orthogonal subfilters, because

$$\hat{q}_i^T \hat{q}_j = \begin{cases} 1 & \text{if } i = j \\ 0.5 & \text{if } |i-j| = 2 \text{ (two adjacent sine or cosine subfilters)} \\ 0 & \text{elsewhere} \end{cases} \quad (31)$$

However, the aim is to make the correlation matrix of the output $\hat{A}_Q$ as diagonally dominant as possible for various inputs. The sidelobes of these filters are small.

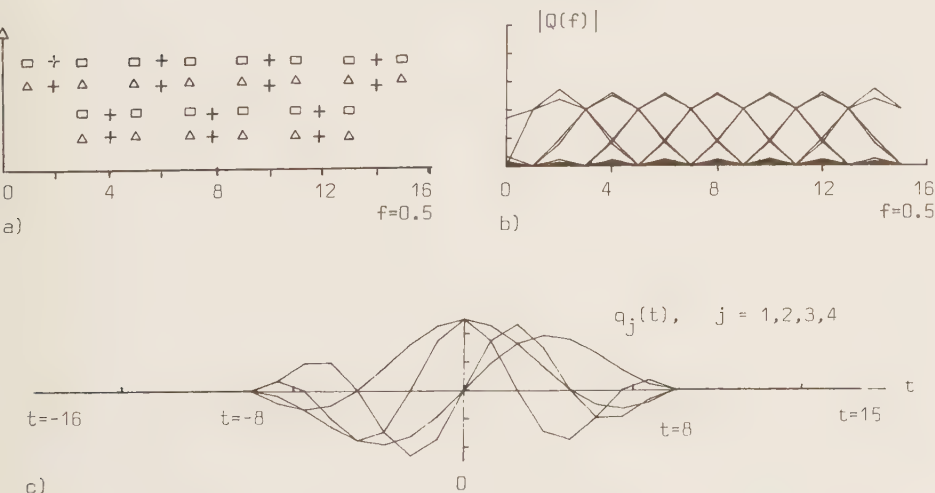


Figure 12. The half circle cosine subfilters, $N=32$.

- a) The centre frequencies.
- b) Actual spectra.
- c) The first four subfilters.

Now, consider again the DFT example in Fig. 8. The filter consists of the lowpass filter $q_0(t)$ frequency translated to the various centre frequencies determined by the DFT. The rectangular shape of $q_0(t)$ in example 1 gives large sidelobes in the frequency domain. These sidelobes will be reduced if a smoother form of the lowpass filter $q_0(t)$ is chosen. A reasonable choice of $q_0(t)$ and $Q_0(f)$ is a triangular shape for $Q_0(f)$. Then, $q_0(t)$ is a $(\sin(x)/x)^2$ time function. The bandwidth of this triangular filter must be at least large enough to locate the truncations at the first zero crossing points in the time domain. This is shown in Fig. 13. The resulting bandpass subfilters, denoted sinc² subfilters, are shown in Fig. 14.

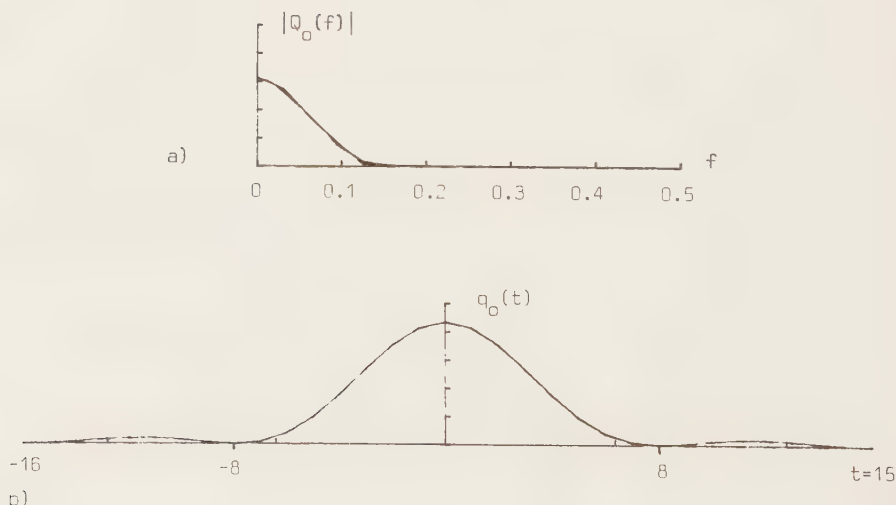


Figure 13. The approximation of triangular bandpass subfilters.

- a) Spectra of the time limited functions $q_0(t)$
 $(q_0(t) = 0 \text{ for } |t| \geq 8)$.
- b) The $(\sin(x)/x)^2$ function.

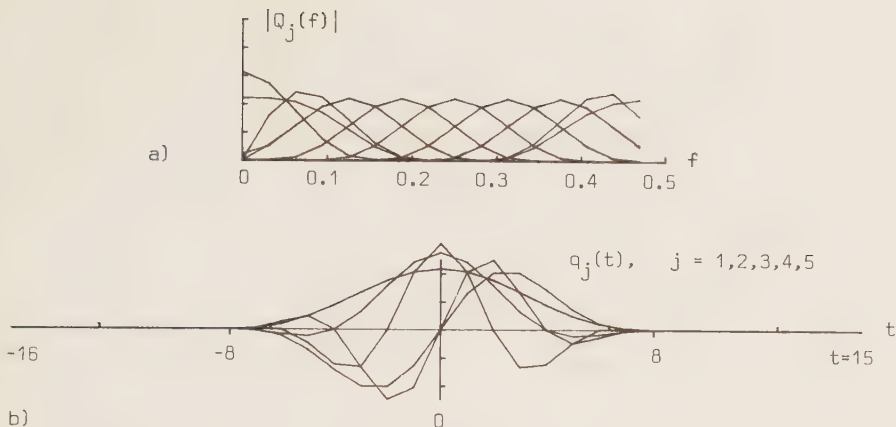


Figure 14. The sinc^2 bandpass subfilters.

a) The spectra.

b) The first five subfilters in the time domain.

The $(\text{sinc}(x)/x)^2$ is a time window with very small sidelobes in the frequency domain while the rectangular time window has large sidelobes. Of course, other choices of $q_0(t)$ may give better compromises between small bandwidth and small sidelobes. A frequently used time window is the Hamming window. In Fig. 15 $q_0(t)$ and $Q_0(f)$ are shown for various time windows.

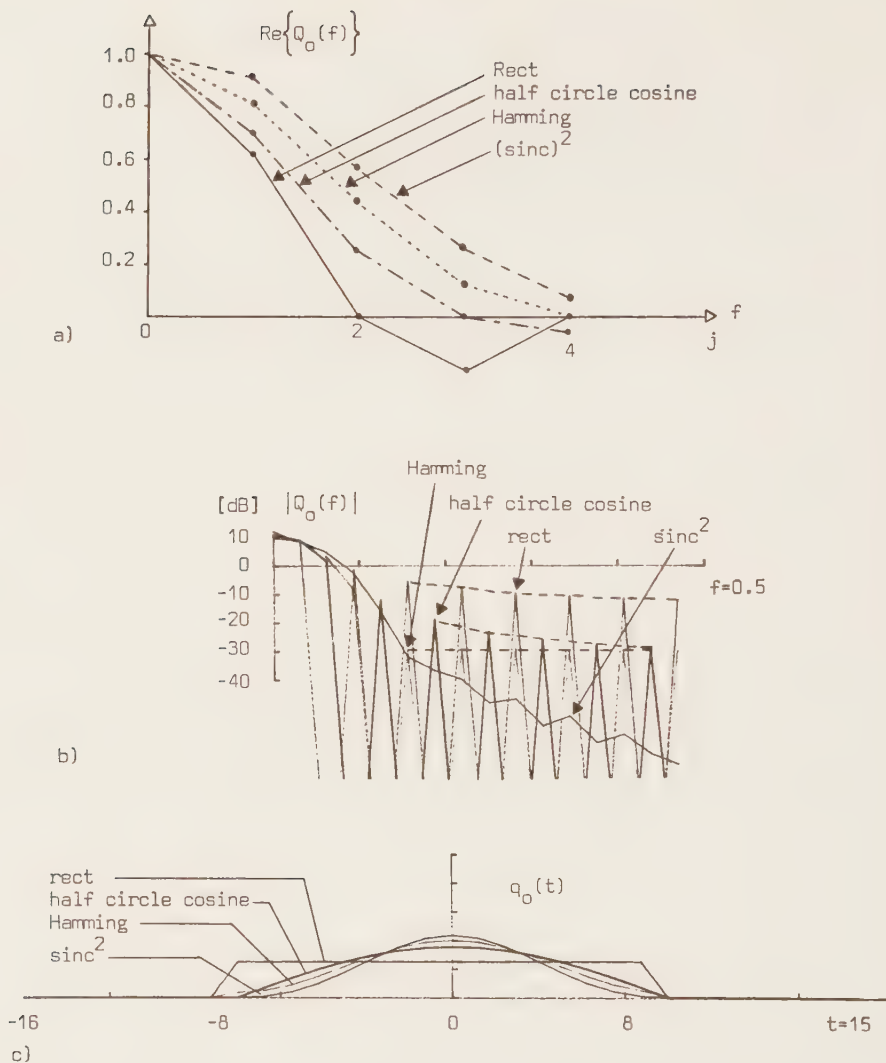


Figure 15. Various $Q_0(f)$ and $q_0(t)$, $N=32$.

- The first five sample points in the frequency domain of $\text{Re}\{Q_0(f)\}$ normalized to $Q_0(f=0) = 1$ (linear scale).
- $|Q_0(f)|$ in a logarithmic scale normalized to $\sum_t q_0(t)^2 = 1$.
- $q_0(t)$ normalized to $\sum_t q_0(t)^2 = 1$.

In the given examples, the number of subfilters and the number of coefficients are of the same order as the filter length $N/2$. But it is not possible to obtain both high resolution and a high rate of convergence. Good approximation in the frequency domain can still be achieved for filters with slowly varying frequency functions if the number of triangular bandpass subfilters (as in Fig. 14) are halved (see Fig. 16). But it must be observed that it is not the magnitude of the spectrum, but the real and imaginary part of the spectrum simultaneously, that are to be approximated. Especially, linear phase components need to be taken into account. If the resulting filter has a large linear phase component, the real and imaginary part of the spectrum vary as cosine and sine, respectively.

The filters can be calculated in the frequency domain. Appendix A presents an algorithm in the frequency domain using a FFT routine. Thanks to the bandpass filter configuration, the calculations can be reduced, see Appendix B. Another effect of the bandpass filter configuration is that filters can be designed with no (or very small) frequency components for say $f \geq f_{\max}$, $f_{\max} < 0.5$, while the impulse response of the filter can still be time-limited.

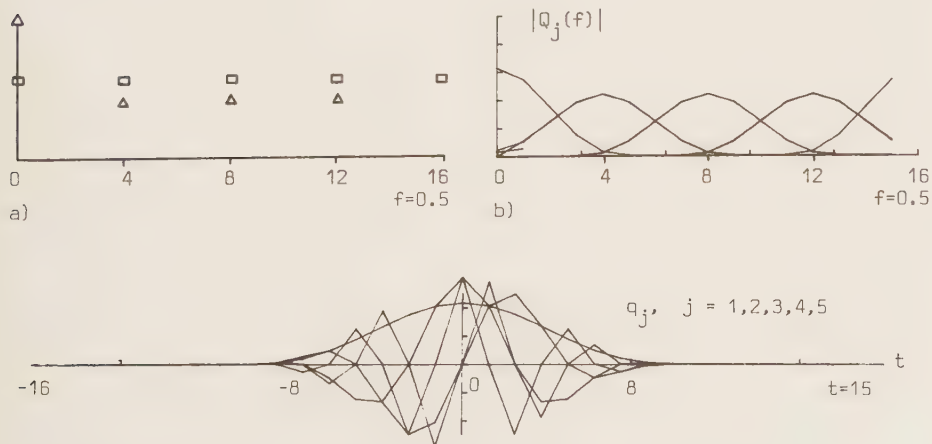


Figure 16. The $N/4 \text{ sinc}^2$ subfilters, $N=32$.

- a) The centre frequencies.
- b) Actual spectra.
- c) The first five subfilters in the time domain.

V. EXAMPLE OF APPLICATION

An example from electromyography analysis (EMG) is used to illustrate the iterative solution of the least squares filters. Details of the example are given in [2]. The input signal (a motor unit potential) is shown in Fig. 17a. The spectrum of this signal is shown in Fig. 17b to a linear scale. In this example the noise level w_0 is set to $w_0=0.1$. Then

$$A(f) = |S(f)|^2 + w_0 \quad (32)$$

is the square of the spectrum of the signal plus noise. This is shown in Fig. 17c.

First, the eigenvalues and eigenvectors of the correlation matrix $\tilde{A}$ in

$$\tilde{A} = (\tilde{S}^T \tilde{S} + w_0 \mathbf{I}) \quad (33)$$

= (11)

are calculated. The signal length and the filter length $N/2$ are 64 sample points. For sufficiently large values of the filter length, the eigenvectors of the matrix $\tilde{A}$ are almost sinusoidal. The eigenvalues are ordered so that

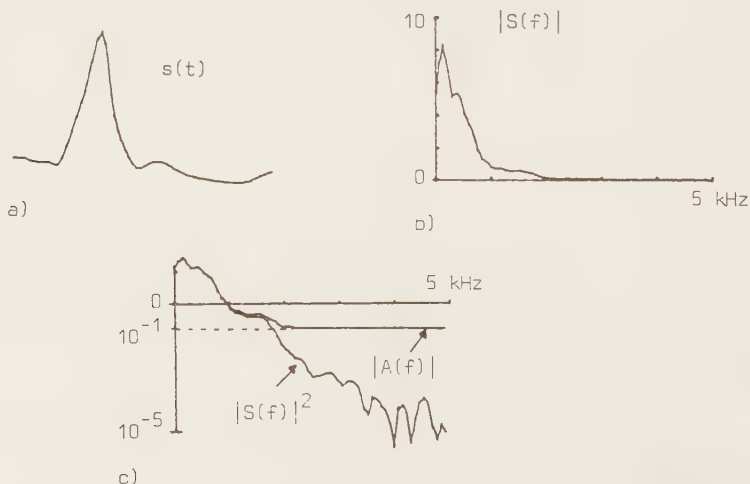


Figure 17. a) An EMG pulse (Motor Unit Potential) $s(t)$, sample rate 10 kHz, $m=64$, signal energy $E_s = \sum_{i=1}^m s_i^2 = 4.8$.
 b) The spectrum of $s(t)$.
 c) $A(f) = |S(f)|^2 + w_0$ with the noise $w_0 = 0.1$.

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n \geq 0 \quad (34)$$

The eigenvalues and eigenvectors are given in Fig. 18, which confirm that the eigenvectors are almost sinusoidal. The largest and smallest eigenvalue are

$$\begin{cases} \lambda_{\max} = 58.7 \\ \lambda_{\min} = 0.1 \end{cases} \quad (35)$$

and the ratio

$$\frac{\lambda_{\max}}{\lambda_{\min}} \approx 600 \quad (36)$$

This indicates a very slow rate of convergence for a steepest descent algorithm. The spectra of the first four $N/2$ length eigenvectors are shown in Fig. 19a, computed by an $N=128$ points DFT.

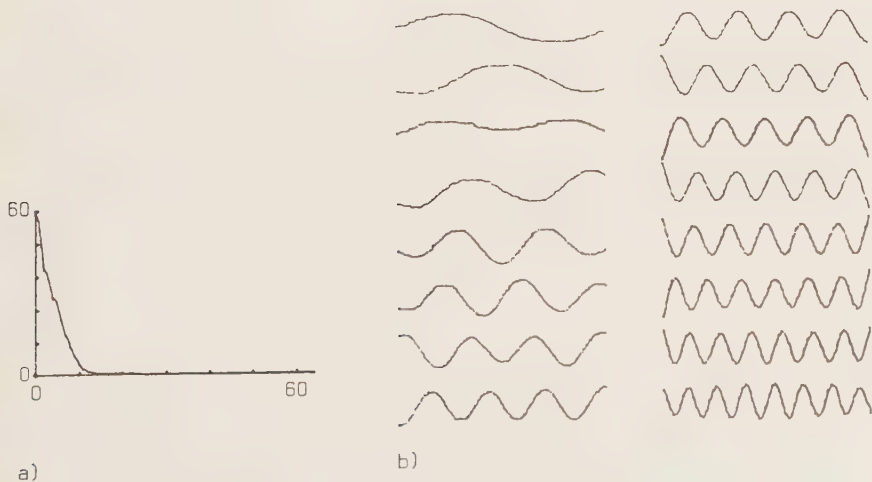


Figure 18. The eigenvalues and the eigenvectors of the matrix $A = \tilde{S}^T \tilde{S} + w_{\Omega} I$ with the signal in Fig. 17, $n=64$.

- The eigenvalues ($\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$).
- The first 16 eigenvectors.

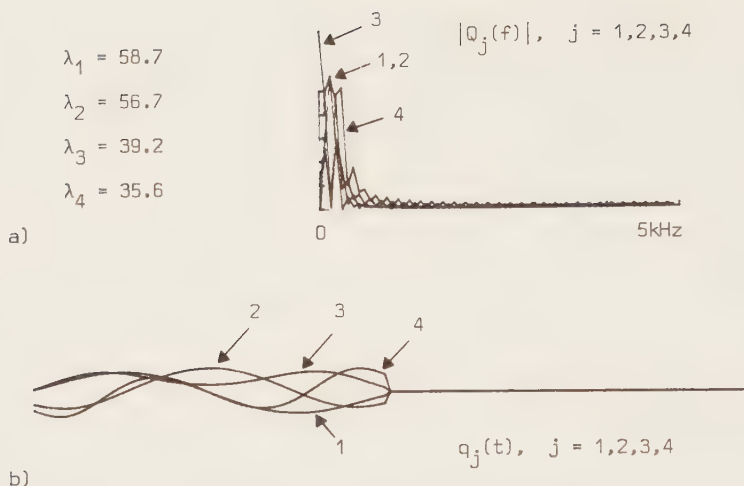


Figure 19. a) The spectra of the first four eigenvectors of the matrix $\underline{A}$ ($N=128$).

b) The eigenvectors in the time domain.

The optimal filter for the signal given in Fig. 17 is shown in Fig. 20. The desired output waveform $g(t)$ is shown in Fig. 20c in the frequency domain and in Fig. 20d in the time domain. It is specified in the frequency domain by a cosine roll off function (Fig. 20c). The resulting filter is shown in Fig. 20a and b. The spectrum of the filter impulse response (Fig. 20b) has high magnitude where the signal spectrum is low (Fig. 17b) up to frequencies about 2.5 MHz. The noiseless output is shown in Fig. 20e.

The original steepest descent algorithm

First, an example will be given of the behaviour of the original steepest descent algorithm (12). The algorithm is written

$$\underline{u}^{k+1} = \underline{u}^k - \alpha' \cdot \frac{2}{\lambda_{\max}} \cdot \underline{x}(\underline{u}^k) \quad (37)$$

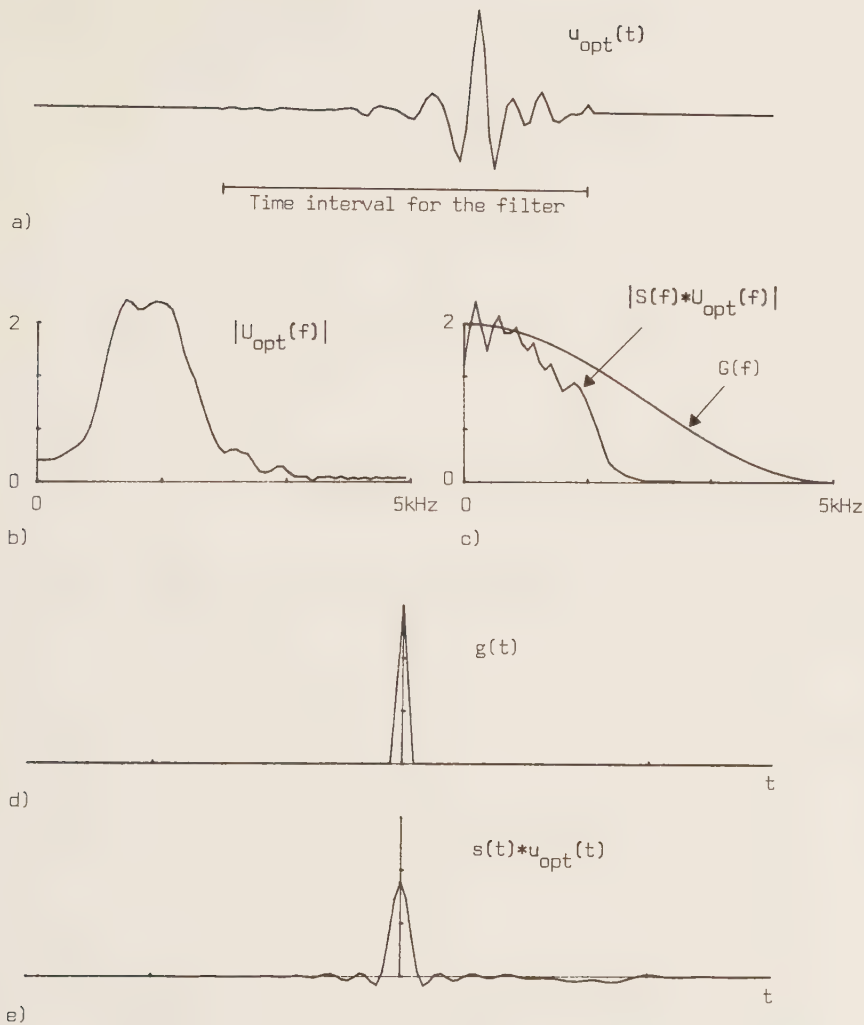


Figure 20. The optimal filter $\underline{u}_{\text{opt}} = \underline{u}^* = \underline{A}^{-1} \underline{S}^T \underline{g}$ ($N=128$).

- a) The impulse response $u_{\text{opt}}(t)$.
- b) The spectrum $U_{\text{opt}}(f)$.
- c) The desired output waveform $G(f)$ and the noiseless output $Y(f) = S(f) * U_{\text{opt}}(f)$.
- d) The desired output waveform $g(t)$.
- e) The noiseless output $y(t) = s(t) * u_{\text{opt}}(t)$.

Convergence of (37) appears for $\alpha' < 1$. However, in most applications the value of $\lambda_{\max}$ is not known. One problem is then to find the value of $\lambda_{\max}$ or an approximation of $\lambda_{\max}$, that is needed for calculating the step size α , where

$$\alpha = \alpha' \cdot \frac{2}{\lambda_{\max}} \quad (38)$$

Bounds of the eigenvalues are [6]

$$a = \max_f \left\{ |S(f)|^2 + w_0 \right\} \geq \lambda_j \geq \min_f \left\{ |S(f)|^2 + w_0 \right\} = b \quad (39)$$

$$j = 1, 2, \dots, N$$

The combination of (39) and the fact that the eigenvectors are almost sinusoidal gives an approximation for $\lambda_{\max}$ and also an approximation of the corresponding eigenvector.

The behaviour of the iterative solution for the step sizes $\alpha'=0.8$ and $\alpha'=1.0$ is shown in Figs. 21 and 22 ($\underline{u}^0 = \underline{0}$). The dashed lines show the optimal filter. For $\alpha'=0.8$ (Fig. 21) convergence is achieved, but since the value of α' is nearly maximum, the solution oscillates around the frequency f_0 (where f_0 is approximately the centre frequency of the first eigenvector). Then for $\alpha'=1.0$ (Fig. 22) oscillation around f_0 gives divergence of the algorithm. Figure 21 shows that the rate of convergence is very low since the eigenvalues corresponding to eigenvectors with higher centre frequencies are very small. Furthermore, the number of iterations needed to reach some point near the solution is quite high.

The conclusion to be drawn from this example is that in problems where $A(f)$ and the resulting filter $U(f)$ have mainly different frequency contents, some sort of amplitude normalization in the frequency domain is very important for the possibility of increasing the rate of convergence.

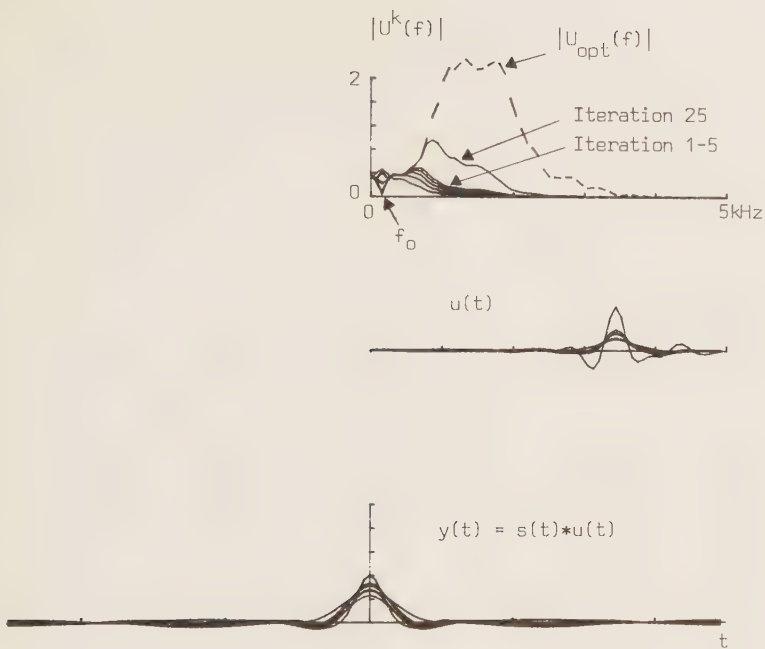


Figure 21. The iteration of the original LS problem ($\alpha' = 0.8$), $\underline{u}^0 = \underline{0}$.

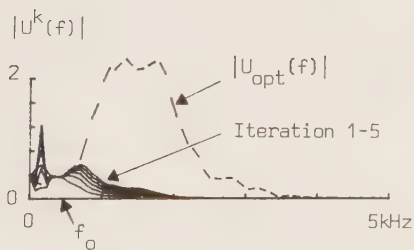


Figure 22. The iteration of the original LS problem ($\alpha' = 1$), only $|U(f)|$.

The DFT and the modified DFT subfilters

The configuration of the filter as bandpass subfilters in parallel enables the spectrum to be normalized. The filter is now written

$$\underline{u} = \sum_{j=1}^N c_j \frac{1}{\sigma_j} q_j \quad \begin{matrix} (40) \\ = (18) \end{matrix}$$

where σ_j is determined from (23).

The values of σ_j^2 for this numerical example of DFT and modified DFT subfilters are shown in Fig. 23. The maximum value of σ_j^2 for the DFT filters is 58.66 for $j=3$. This is roughly the same as the value of $\lambda_{\max}$ in (35). Compare also the first eigenvectors in Fig. 18 with the third DFT subfilter in Fig. 9. The values of σ_j^2 for the cosine subfilters for large j (even j in Fig. 23a) are 0.17 while w_0 is only 0.1. This is due to the large sidelobes in the DFT subfilters. The values of σ_j^2 for the modified DFT subfilters are of about the same order as for the DFT subfilters. But now for large values of j both the sine and cosine subfilters have $\sigma_j^2 \approx w_0$. The values of $\lambda_{\max}$ and $\lambda_{\min}$ for the new correlation matrices $\underline{A}_Q$ (22) are now

$$\begin{array}{lll} \text{DFT:} & \lambda_{\max} = 18.9 & \lambda_{\min} = 0.42 \\ \text{modified DFT:} & \lambda_{\max} = 9.2 & \lambda_{\min} = 0.42 \end{array} \quad (41)$$

The behaviour of the steepest descent algorithm (12) using (40) and (21) is shown in Figs. 24 and 25 for step size $\alpha' = 0.8$. Five iterations are shown ($\underline{c}^0 = \underline{0}$). The normalizations in the frequency domain have increased the rate of convergence (compare with Fig. 21), but several iterations are still needed to reach a point near the optimum (dashed lines).

The half circle cosine and the Hamming subfilters

Another way of approaching the optimal curve is found from the third example of $\underline{Q}$, the half circle cosine (HCC) subfilters. In this case, $\lambda_{\max}$ and $\lambda_{\min}$ are

$$\lambda_{\max} = 1.92 \quad \lambda_{\min} = 0.01 \quad (42)$$

DFT			Modified DFT		
Index	cosine subfilter	sine subfilter	Index	cosine subfilter	sine subfilter
1	37.9423				
2,3	54.1564	58.6583	2,3	43.44	58.6583
4,5	28.7732	33.6476	4,5	48.888	33.6476
.	22.7732	26.1313	.	29.465	26.1313
.	11.6648	13.9446	.	19.3421	13.9446
.	4.20408	5.7141	.	9.63186	5.7141
.	1.50338	2.39845	.	3.44152	2.39845
.	.755122	1.33472	.	1.72335	1.33472
.	.574651	.974021	.	1.13976	.974021
.	.504943	.785251	.	.848256	.785251
.	.456346	.658228	.	.733305	.658228
.	.339477	.483187	.	.569879	.483187
.	.246029	.345129	.	.407771	.345129
.	.203716	.268352	.	.298276	.268352
.	.191508	.229531	.	.246988	.229531
.	.186077	.203037	.	.215485	.203037
.	.181986	.181984	.	.191611	.181984
.	.180838	.167006	.	.173992	.167006
.	.180343	.155114	.	.160812	.155114
.	.179079	.144376	.	.149594	.144376
.	.178483	.135868	.	.139793	.135868
.	.178043	.128775	.	.132248	.128775
.	.177383	.122522	.	.125511	.122522
.	.177187	.117628	.	.119919	.117628
.	.176989	.113481	.	.115483	.113481
.	.176809	.110007	.	.111662	.110007
.	.176736	.107216	.	.108524	.107216
.	.176656	.104925	.	.106028	.104925
.	.176592	.103116	.	.103942	.103116
.	.176636	.101837	.	.102438	.101837
.	.176522	.10078	.	.101266	.10078
62,63	.176481	.100198	62,63	.100432	.100198

Figure 23. The values of normalization parameter σ_j^2 for the DFT and modified DFT subfilters ($j = 1, 2, \dots, 63$ and $j = 2, 3, \dots, 63$).

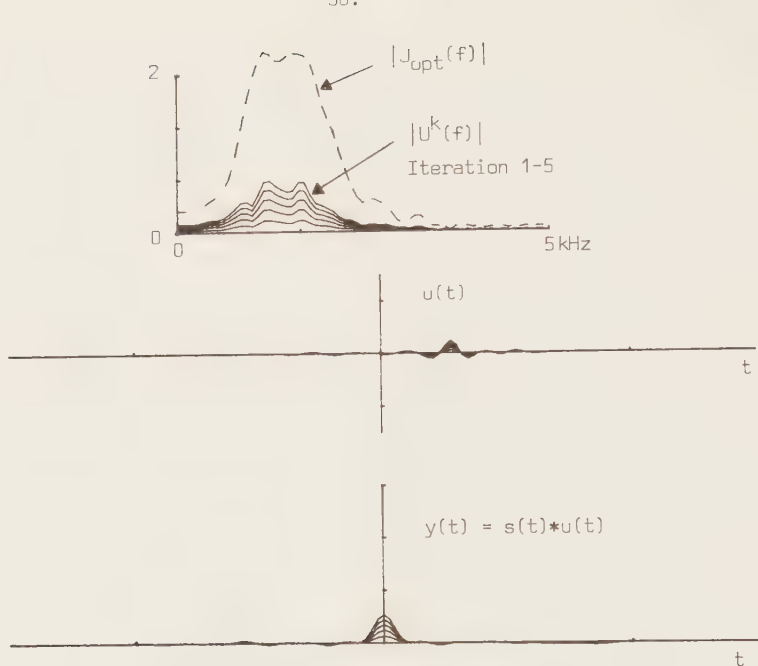


Figure 24. The iteration using the DFT subfilters, $\alpha' = 0.8$.

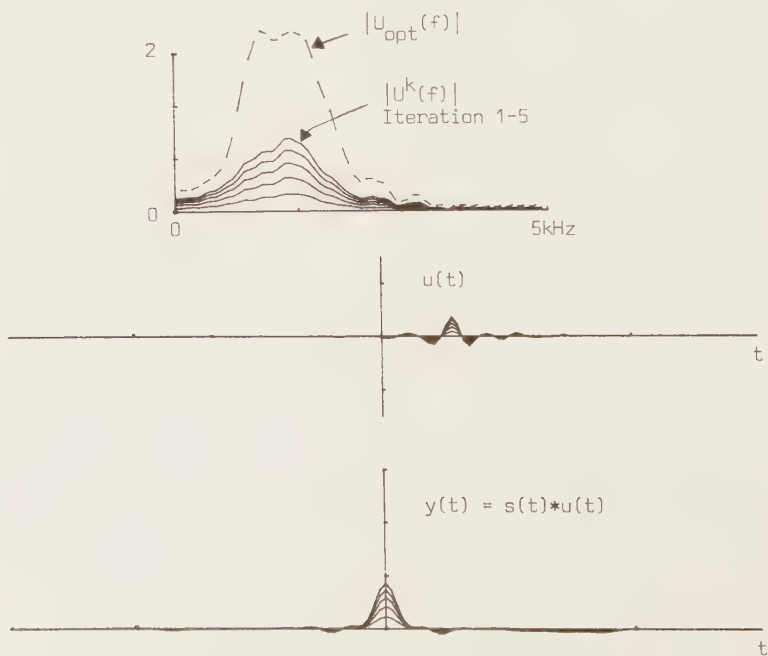


Figure 25. The iteration using the modified DFT subfilters, $\alpha' = 0.8$.

but $\lambda_{\min}$ is close to zero. It is also assumed that the exact value of $\lambda_{\max}$ is not known. Then the algorithm is

$$\underline{c}^{k+1} = \underline{c}^k - \alpha \underline{L}(\underline{c}) \quad (43)$$

where α' and $2/\lambda_{\max}$ cannot be separated. If $\lambda_{\max} = 2$, a value of $\alpha' = 0.8$ corresponds to $\alpha = 0.8$. The first three iterations for $\alpha = 0.8$ are shown in Fig. 26. A usable filter has been reached already after the first iteration. This is due to the sidelobes of the HCC subfilters being small, see Figs. 12 and 15. The rate of final adjustment to the optimal curve in Fig. 26 is low. This is due to the optimal filter having a large linear phase component. Then the real and imaginary parts of the spectra vary as cosine and sine, respectively. The real and imaginary parts of the spectrum are shown in Fig. 27 for these iterations.

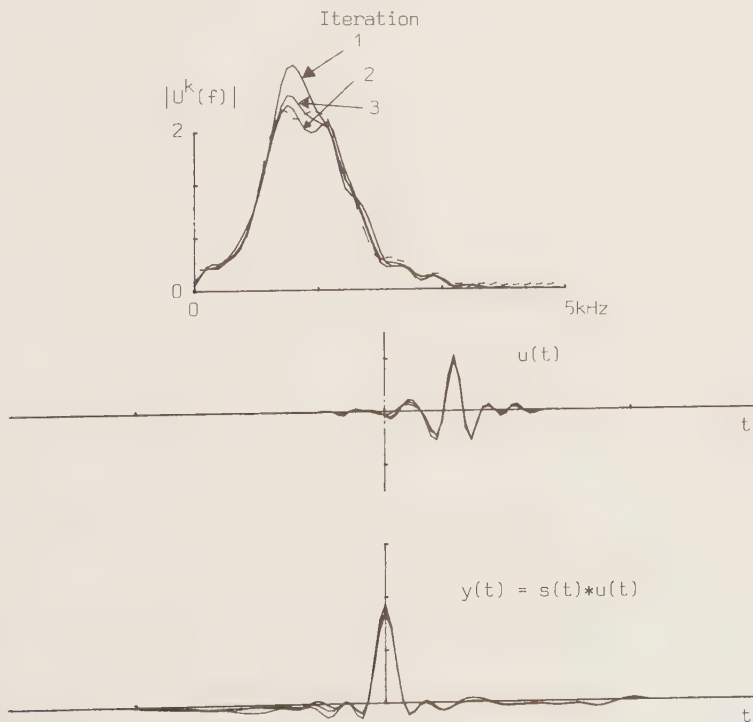


Figure 26. The iteration using the HCC subfilters, $\alpha = 0.8$
 $(\alpha' = \alpha \cdot 1.92/2 = 0.77)$.

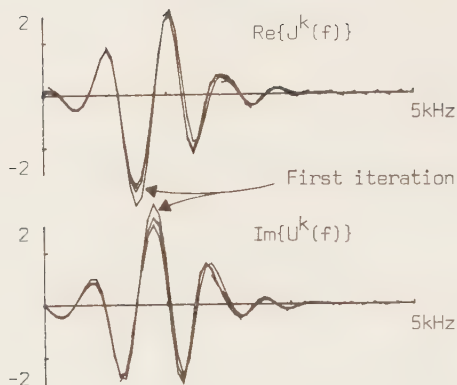


Figure 27. The real and imaginary parts of the filter in Fig. 26.
The linear phase component affects the filter so that the real and imaginary parts oscillate.

Finally, the iterations using the Hamming time window are shown in Fig. 28. This behaviour is about the same as for the HCC filters. But the final adjustment needs more iterations than are needed by the HCC filters due to the mainlobe being wider, see Fig. 15.

The four different subfilters presented in this section can be divided into two groups. The first group consists of the DFT and modified DFT subfilters and the second group of the HCC and Hamming subfilters. The subfilters in the first group have small mainlobes and large sidelobes. The subfilters in group two have wider mainlobes but smaller sidelobes. The smaller sidelobes enable the algorithm to reach usable filters already after the first iteration. However, the final adjustment can be difficult. This can also be seen as an approximation problem, i.e. how well the filter can be approximated by the subfilters.

The trade-off between the rate of final adjustment and the number of iterations needed to find usable filters must be taken into account in the choice of subfilters. However, the final adjustment is in many cases not too important and the HCC subfilters are usually found to be suitable.

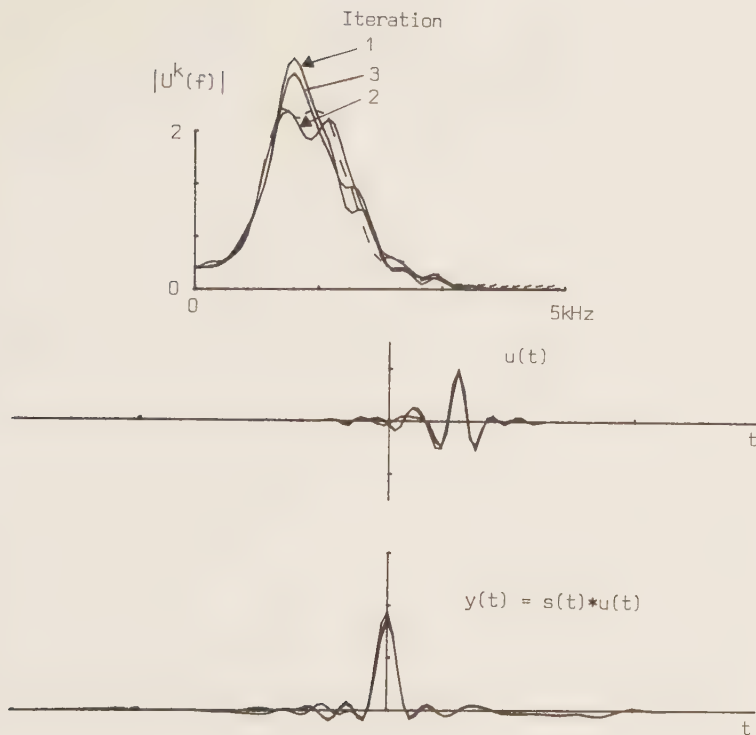


Figure 28. The iteration using the Hamming time window, $\alpha=0.8$.

The rate of convergence

The examples presented in this section show a very slow rate of convergence for the steepest descent algorithm. About 3000 iterations were needed to determine the optimal filter (with an accuracy of about 5 digits). The configuration of the filter as bandpass subfilters in parallel increased the rate of convergence. With the DFT and the modified DFT subfilters the number of iterations ($\alpha'=0.8$) needed to attain the same accuracy were 180 and 90, respectively. The HCC filter reached a point near the optimum filter in a few iteration steps. But after that the value of the cost function decreased very slowly and the algorithm never actually reached the same value of the

minimal cost as the DFT and the modified DFT subfilters did. The error of the filter in step k

$$||\underline{e}^k||^2 = ||\underline{u}^k - \underline{u}^*||^2 \quad (44)$$

is shown in Fig. 29.

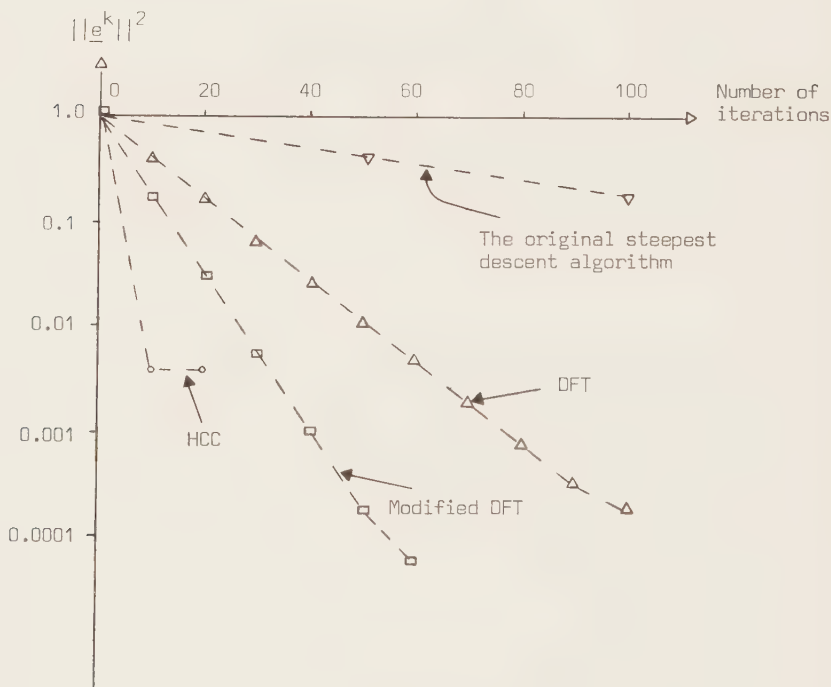


Figure 29. The actual rates of convergence for the examples used in this section, $\alpha'=0.8$.

Band limited FIR filters

The configuration of the filter as bandpass subfilters in parallel makes it possible to design FIR filters with impulse responses that have most of the energy in desired frequency bands. Consider again the structure of the filter $u(t)$,

$$u(t) = \sum_{j=k_1}^{k_2} c_j \frac{1}{\sigma_j} q_j(t) \quad (45)$$

This time, only a limited number of subfilters are used. Then a desired frequency band is specified by k_1 and k_2 . Consider the half circle cosine (HCC) filters shown in Fig. 12. The centre frequencies of the subfilters are

$$f_k = 2 \cdot k \cdot \frac{1}{N} \cdot F_s \quad k = 1, 2, \dots, \frac{N}{4} - 1 \quad (46)$$

where F_s is the sample rate.

The signal $s(t)$, Fig. 17, has very little energy at high frequencies. Then the frequency range of the filter can be restricted. Figure 30 shows an example where only 20 subfilters (10 cosine and 10 sine) are used. Figure 30a shows the spectrum and the spectrum of the optimum filter. The spectra of the noiseless outputs are shown in Fig. 30b. The range of the centre frequencies (46) is

$$\frac{1}{64} \cdot F_s \leq f_k \leq \frac{10}{64} \cdot F_s \quad (47)$$

that is

$$0.16 \leq f_k \leq 1.6 \quad (\text{MHz}) \quad (48)$$

The noiseless outputs are shown in Fig. 30c and d. The output from the band limited filter, Fig. 30c, differs only slightly from the output from the optimal filter, Fig. 30d.

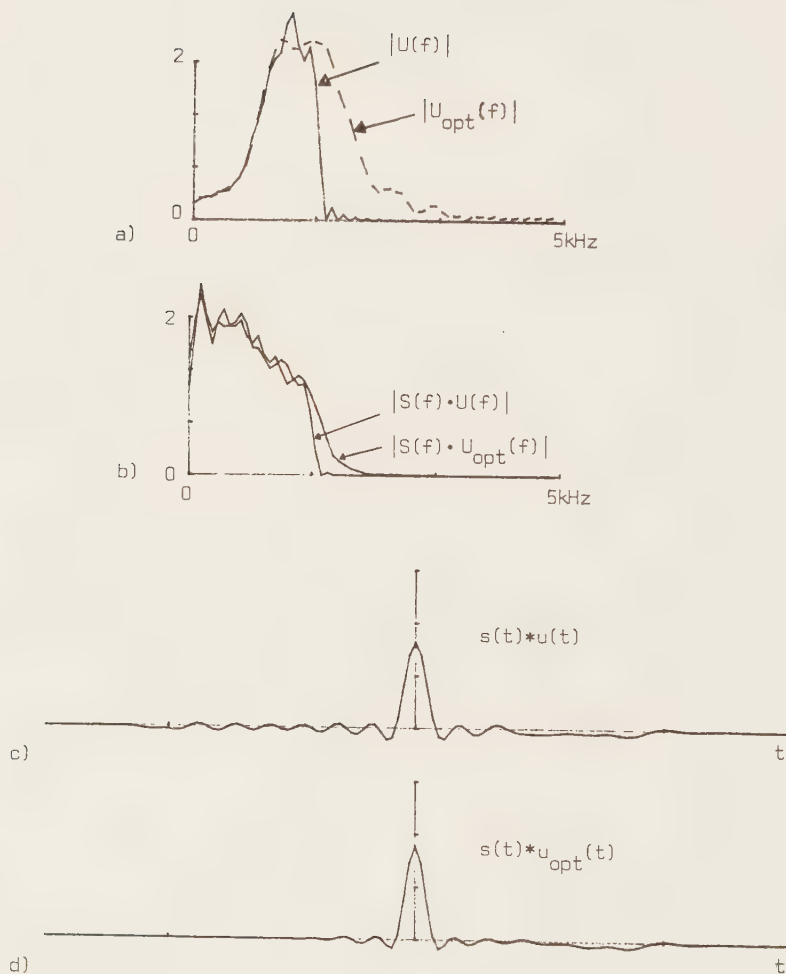


Figure 30. An example with a smaller number of subfilters (10 cosine and 20 sine subfilters).

- a) The spectra. The dashed line is the optimum filter.
- b) The spectra of the noiseless outputs.
- c) The output from the filter.
- d) The output from the optimum filter.

VI. DISCUSSION AND CONCLUSIONS

The structure of bandpass subfilters in parallel is found to be a suitable configuration for improving the rate of convergence for iterative solution schemes. An important property is that the eigenvectors of the correlation matrix become increasingly sinusoidal as the length of the filter is increased. In the example presented in Section V, the number of iterations was reduced from about 3000 for the original steepest descent algorithm to 180 and 90 by using the DFT and modified DFT subfilters respectively, without sacrificing accuracy. The half circle cosine and the Hamming subfilters reached a point near the optimum after only one iteration.

The number of bandpass subfilters is of the same order as the length of the filter. This is due to the fact that the filter is assumed to be time-limited. Of course, it is possible to increase the rate of convergence even further by increasing the length of the bandpass subfilters. Then for the same number of subfilters the interval between the centre frequencies are increased. This will give more diagonally dominant correlation matrices, see Section III.

The bandpass filter configuration can also be used in order to reduce the amount of computation. The calculations are performed in the frequency domain, as described in Appendix A. Appendix B shows how the amount of calculation can be reduced.

A third and very important feature of this configuration is that it enables a time-limited filter to be designed with the transfer function (spectrum) restricted to a given frequency band. This can be done by using only those bandpass subfilters that have centre frequencies in this band.

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to Ass. Prof. Göran Salomonsson for many helpful discussions.

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APPENDIX A

Computation of the filters in the frequency domain

The bandpass subfilter configuration is well suited for calculations in the frequency domain. Parseval's theorem states that

$$\sum_{t=1}^N x(t) y(t) = \frac{1}{N} \sum_{k=0}^{N-1} X(f_k) Y(f_k)^* \quad (A1)$$

and

$$\sum_{t=1}^N x(t)^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(f_k)|^2 \quad ; \quad f_k = \frac{k}{N} \quad (A2)$$

where $X(f)$, $Y(f)$ are the DFT of $x(t)$, $y(t)$, respectively. The sample rate is normalized to 1. Now, using (A1), the cost function (20) can be written

$$J_f(\underline{c}) = \sum_f \left\{ |S(f_k) U(f_k) - G(f_k)|^2 + w_0 |U(f_k)|^2 \right\} \quad (A3)$$

with

$$U(f_k) = \sum_{j=1}^{N_c} c_j \frac{1}{\sigma_j} Q_j(f_k) \quad ; \quad j = 1, N_c \quad (A4)$$

The summation over f means that k in f_k takes the values $k = 0, 1, 2, \dots, N-1$. The factor $1/N$ is also dropped.

The definition of the time intervals, Fig. 7, guarantees that the convolutions will be noncircular. The input signal $s(t)$, the filter $u(t)$ and the subfilters $q_j(t)$ are all time-limited to $-N/2 \leq t < N/2$. Inserting (A4) in (A3) gives

$$\begin{aligned} J_f(\underline{c}) = \sum_f \left\{ \left[S \cdot \sum_j c_j \frac{1}{\sigma_j} Q_j - G \right]^* \left[S \cdot \sum_j c_j \frac{1}{\sigma_j} Q_j - G \right] + \right. \\ \left. + w_0 \sum_j c_j \frac{1}{\sigma_j} Q_j^* \cdot \sum_j c_j \frac{1}{\sigma_j} Q_j \right\} \quad (A5) \end{aligned}$$

Differentiating (A5) with respect to $\underline{c}$ gives

$$\underline{\ell}(\underline{c}) = \frac{1}{2} \nabla_{\underline{c}} J_f(\underline{c}) \quad (A6)$$

with

$$\left(\underline{\ell}(\underline{c})\right)_j = \frac{1}{2} \frac{dJ_f(\underline{c})}{dc_j} = \sum_f \left\{ S^* \frac{1}{\sigma_j} Q_j^* (S U - G) + w_o \frac{1}{\sigma_j} Q_j^* U \right\} \quad (A7)$$

where (A4) is also used. Further simplification of (A7) gives

$$\begin{aligned} \left(\underline{\ell}(\underline{c})\right)_j &= \sum_f S^* \frac{1}{\sigma_j} Q_j^* E_y + w_o \frac{1}{\sigma_j} Q_j^* U = \\ &= \sum_f \frac{1}{\sigma_j} Q_j^* E \end{aligned} \quad (A8)$$

with

$$E_y = S U - G \quad (A9)$$

and

$$E = S^* E_y + w_o U \quad (A10)$$

Then the algorithm can be summarized in the following steps:

1. Choose initial value $\underline{c}^0 = \underline{0}$
- LOOP
2. Compute the filter $U(f) = \sum_{j=1}^{N_c} c_j \cdot \frac{1}{\sigma_j} \cdot Q_j(f)$
3. Compute the DFT of the input $S(f) = \text{FT}(s(t))$
4. Compute the output $Y(f) = S(f) \cdot U(f)$
5. Inverse DFT $y(t) = \text{IFT}(Y(f))$
6. Compute the output error $e_y(t) = y(t) - g(t)$
7. Compute $E_y(f) = \text{FT}(e_y(t))$
8. Compute $E(f) = S(f)^* E_y(f) + w_o U(f)$
9. Correlate $j = 1, 2, \dots, N_c$ $\left(\underline{\ell}(\underline{c})\right)_j = \sum_f \frac{1}{\sigma_j} Q_j^* E(f)$
10. Update $\underline{c}$ $\underline{c}^{k+1} = \underline{c}^k - \alpha \underline{\ell}(\underline{c})$
11. Return to step 2.

If the subfilters $q_j(f)$ are the eigenvectors of the matrix $\underline{A}_Q$, $Q_j(f)$ will be complex with normally both the real and imaginary parts not equal to zero. But $q_j(t)$ is normally chosen so that $Q_j(f)$ is either real or imaginary. Then

$$\begin{aligned} \left(\underline{x}(\underline{c}) \right)_j &= \sum_f \frac{1}{\sigma_j} Q_j^* \left(\text{Re}\{E\} + j \text{Im}\{E\} \right) = \\ &= \sum_f \frac{1}{\sigma_j} \begin{cases} Q_j \text{Re}\{E\} & \text{if } Q_j \text{ real} \\ Q_j \text{Im}\{E\} & \text{if } Q_j \text{ imaginary} \end{cases} \end{aligned}$$

Furthermore the summation in the frequency domain needs only be done in the interval $0 \leq f \leq 0.5$.

APPENDIX B

The amount of computation for the algorithm in Appendix A may be further reduced by using the fact that Q_j , $j = 1, 2, \dots, N$ are narrow-banded bandpass filters. In step 2

$$U(f) = \sum_{j=1}^{N_c} c_j \frac{1}{\sigma_j} Q_j(f) \quad (B1)$$

and in step 9

$$\left(\frac{\ell(c)}{\sigma} \right)_j = \sum_f \frac{1}{\sigma_j} Q_j^* E(f) \quad (B2)$$

the calculations are proportional to N^2 , i.e. both the subscript j and the argument f vary in the interval $1, 2, \dots, N$. But for the narrowband filters, the summation over f can be made over say 3-5 points around the centre frequencies. Especially, the sinc^2 and the half circle cosine filters are below -30 dB at five points away from the centre frequency, see Fig. 15. Truncation implies approximation and the effect of this in the time domain is that the resulting filters need not necessarily be completely time-limited. However the energy of the filter outside the defining interval is negligible.

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DIGITAL FILTERING OF ULTRASONIC ECHO SIGNALS

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DIGITAL FILTERING OF ULTRASONIC ECHO SIGNALS

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ABSTRACT

The axial resolution of the ultrasonic pulse echo method is strongly dependent on the duration of the impulse response of the ultrasonic transducer. In this paper the application of digital pulse shaping filters to decrease the duration of the ultrasonic echo pulses is described. The filter that is used is the so called weighted least squares filter. It is shown that axial resolution is improved by using this filter. It is also shown that the filter can be used to preshape the transmitted ultrasonic signal.

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II. BACKGROUND

III. METHOD

IV. PRESHAPING

V. EXPERIMENTS

VI. DISCUSSION AND CONCLUSIONS

REFERENCES

I. INTRODUCTION

The ultrasonic pulse echo method is widely used in medical diagnosis. The theory and the biomedical applications of ultrasound are described in [10] and [11]. The ultrasonic pulse echo method is based on the reflection of ultrasonic energy from surfaces between media of different acoustic impedance. The time delay between emission of a pulse and reception of an echo is a measure of the distance from the transducer to the reflecting surface. Usually, the same transducer is used for both emission and reception. Sound is propagated in water and in soft human tissue at about 1500 m/s and an echo delay of one microsecond corresponds to a distance of 0.75 mm.

The axial resolution of the ultrasonic pulse echo method is strongly dependent on the duration of the transmitted ultrasonic pulses. Owing to the narrow bandpass characteristic of the ultrasonic waveform this duration is rather long. The development of high speed A/D and D/A converters now makes it possible to apply digital signal processing techniques to ultrasonic signals. This paper describes how digital filters can be used to decrease the duration of ultrasonic pulses and thereby improve axial resolution.

The frequency range of the ultrasonic signals used in most medical applications is approximately 1 MHz to 5-10 MHz. The repetition rate of the transmitted pulses is usually 50 Hz - 10000 Hz.

Section II of this paper gives the background to the ultrasonic pulse echo method. Section III describes how the digital filters are applied to the improvement of resolution, while preshaping of the transmitted ultrasonic pulses is dealt with in Section IV. Finally, experimental results are presented in Section V.

II. BACKGROUND

The ultrasonic pulse echo method is illustrated in Fig. 1. A short pulse excites the ultrasonic transducer. The transducer generates an acoustic signal in the medium 1. The acoustic wave is then transmitted through the medium 1. At the surfaces between media of different acoustic impedance echo signals are generated. The amplitude of these echoes depends on the difference in the acoustic impedance. Using the notation z_j for the acoustic impedance in medium j the echo reflected from the surface ij is

$$\text{echo amplitude} = \frac{z_j - z_i}{z_j + z_i} \text{ of the transmitted amplitude} \quad (1)$$

The sign of the echo amplitude depends on which of the impedances is the larger.

The media are assumed to be homogeneous and lossless. It is also assumed that the transmitted signal is a plane wave. Many media have roughly the same impedance and the echoes are consequently of very low amplitude. The amplitude of the transmitted signal is then assumed to remain constant as it continues through the media.

When the echo reaches the transducer, a corresponding electrical signal is generated. This received echo signal is assumed to be

$$r(t) = \sum_j a_j s(t - \theta_j) + n(t) \quad (2)$$

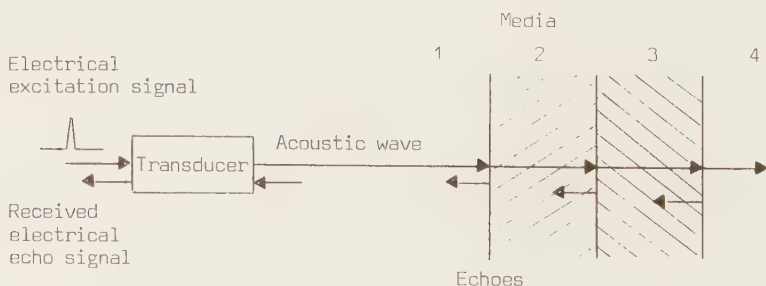


Figure 1. The ultrasonic pulse echo method applied to a system of layered media.

where a_j denotes the amplitude and θ_j the delay of the echo waveform $s(t)$ from surface j . The noise $n(t)$ can be assumed to be white (most noise originates in the receiver amplifier).

Let the sequence of echoes be characterized by

$$h(t) = \sum_j a_j \delta(t - \theta_j) \quad (3)$$

with $\delta(t)$ defined as

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases} \quad (4)$$

(assuming sampled signals). Then the received signal can be written

$$r(t) = h(t) * s(t) + n(t) \quad (5)$$

where $*$ denotes convolution. Equation (5) shows that an equivalent model is that $h(t)$ is assumed to be the input signal to the linear system $s(t)$ as shown in Fig. 2.

The ultrasonic transducer is assumed to be characterized by its impulse response $s(t)$, i.e. the echo from a plane surface perpendicular to the direction of transmission. Unfortunately the duration of $s(t)$ is rather long due to the bandpass characteristic of the transducer impulse response [5] and it will not be possible to resolve surfaces in close proximity. Increased resolution can be achieved by decreasing the duration of the impulse response $s(t)$. In this paper it is shown that digital signal processing techniques can be applied to ultrasonic signals in order to improve axial resolution.

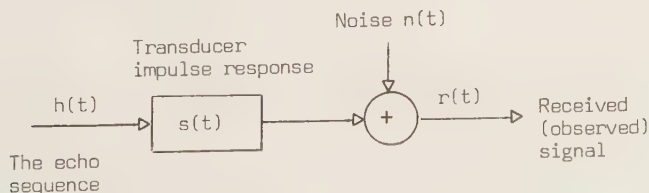


Figure 2. The equivalent model of the received signal. The echo signal is described by $h(t)$ and the transducer by its impulse response $s(t)$.

III. METHOD

The received signal is assumed to be described by delayed echo pulses, equation (5). The method used in this paper to improve axial resolution is to design a filter as shown in Fig. 3. The received signal is passed through this filter before being displayed (or further processed). The filter is designed to reduce the duration of the echo from a single surface.

The notations are defined in Fig. 4. The impulse response of the transducer is denoted $s(t)$ as above (the echo signal from a plane surface including both transmitting and receiving characteristics) and the impulse response of the filter is denoted $u(t)$. The output echo signals from the filter are defined

$$y(t) = s(t) * u(t) \quad (6)$$

and

$$y_n(t) = s(t) * u(t) + n(t) \quad (7)$$

where $y(t)$ is the noiseless output signal and $y_n(t)$ is the output signal including the noise $n(t)$. The filter that reconstructs the echo sequence (3), i.e. the filter that gives

$$y(t) = \delta(t) \quad (8)$$

is the inverse filter.

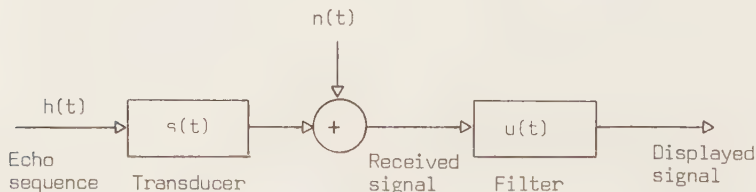


Figure 3. The application of the receiver filter $u(t)$. The filter is designed to improve axial resolution by decreasing the duration of the echoes.

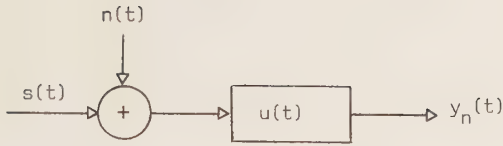


Figure 4. The notations used in the filter design.

$$y_n(t) = s(t) * u(t) + n(t)$$

$$y(t) = s(t) * u(t)$$

However, it is impossible to design the pure inverse filter. Denote the Fourier transform of $s(t)$, $u(t)$ and $y(t)$ with $S(f)$, $U(f)$ and $Y(f)$. Then (6) can be written

$$Y(f) = S(f) U(f) \quad (9)$$

and the inverse filter $U_{inv}(f)$ is

$$U_{inv}(f) = \frac{1}{S(f)} \quad (10)$$

The inverse filter (10) is now modified by defining a desired output waveform $g(t)$ ($G(f)$) that gives

$$U_{inv}(f) = \frac{G(f)}{S(f)} \quad (11)$$

The desired output $G(f)$ in (11) must be chosen so that $U_{inv}(f)$ can be achieved. A different approach is to use the least squares pulse shaping filter that is defined [8] as

$$U_{LS}(f) = \frac{|S(f)|^2}{|S(f)|^2 + R_n(f)} \cdot \frac{G(f)}{S(f)} \quad (12)$$

where $R_n(f)$ is the noise power spectrum. However neither the inverse filter (11) nor the least squares filter (12) are suitable for ultrasonic signals due to the bandpass characteristic of the impulse response of the transducer. Alternative formulations of the least squares filters are found in [8] and [1]. Here the weighted least squares filter [1] will be used.

The weighted least squares (WLS) filter is designed to make the pulse duration in the output $y(t)$ shorter than in the input $s(t)$. This filter must satisfy the following three conditions:

1. The output must have a high amplitude at one point, say for $t=0$.
2. The amplitude of the output must be low outside some resolution interval, say for $|t| > T$.
3. The output noise must be low.

Combining (1), (2) and (3) with the expected noise power w_0 , the weighted least squares filter is defined as the filter that minimizes the cost function

$$J = \left(1 - s(t) * u(t) \Big|_{t=0}\right)^2 + \sum_{|t| > T} \left(s(t) * u(t)\right)^2 + w_0 \sum_t \left(u(t)\right)^2 \quad (13)$$

The cost function is illustrated in Fig. 5. The dashed lines represent a time window in which the output may take arbitrary values except at $t=0$ where the output is desired to be 1. Outside this window the output must be low.

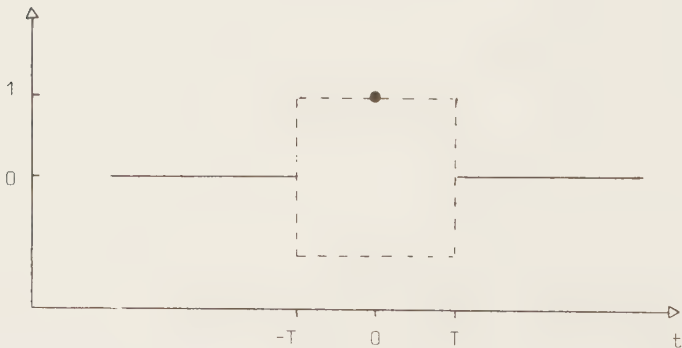


Figure 5. The time window that illustrates the cost function for the WLS filter. At $t=0$ the desired output is 1 and for $|t| > T$ the desired output is small. Elsewhere the output may take arbitrary values.

By defining a weighting function $w(t)$ that is equal to zero for $|t| \leq T$, $t \neq 0$, and equal to one elsewhere, i.e.

$$w(t) = \begin{cases} 0 & |t| \leq T, \quad t \neq 0 \\ 1 & t = 0 \quad \text{or} \quad |t| > T \end{cases} \quad (14)$$

the cost function (13) can be written

$$J = \sum_t \left\{ w(t) \cdot [s(t) * u(t) - \delta(t)] \right\}^2 + w_0 \sum_t (u(t'))^2 \quad (15)$$

In (15) two parameters have to be set by the designer. These are the resolution parameter T and the parameter w_0 that takes into account the input noise. But these parameters do not affect the filter independently of each other [1].

An important feature of the weighted least squares filter is that it is related to both the pure inverse (time-limited) filter and the matched filter. Indeed both the pure inverse filter and the matched filter are special cases of the weighted least squares filter [1]. The pure inverse filter is found by letting both w_0 and T be equal to zero. The matched filter is found by choosing a large value of either T or w_0 .

Assuming a frequency spectrum of a transducer impulse response as shown in Fig. 6a, the frequency spectra of the inverse filter and the matched filter impulse responses will look like the dotted curves in Fig. 6b. The matched filter is the filter that maximizes the signal/noise ratio but unfortunately it also increases the pulse duration. A suitable filter must have an impulse response with a spectrum with roughly the shape given by the solid line in Fig. 6b. The weighted least squares filter has this property.

Examples of transducers and filters

The impulse responses of two different transducers A and B for echoes reflected from a plexiglass surface in water are shown in Fig. 7. Transducer A is a conventional transducer and has a rather narrowbanded impulse response, Fig. 7b. Transducer B has been heavily damped to give short echo pulses [5]. The spectra

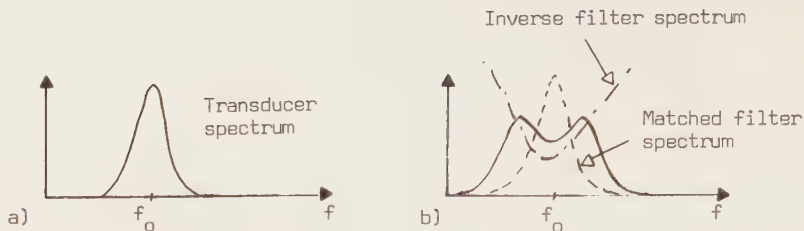


Figure 6. Expected spectra of the impulse response of a suitable filter.

- a) Spectrum of the transducer impulse response.
- b) Spectra of the inverse filter and the matched filter impulse responses (dotted lines) and an approximately shape for a filter suitable for decreasing the duration of ultrasonic pulses.

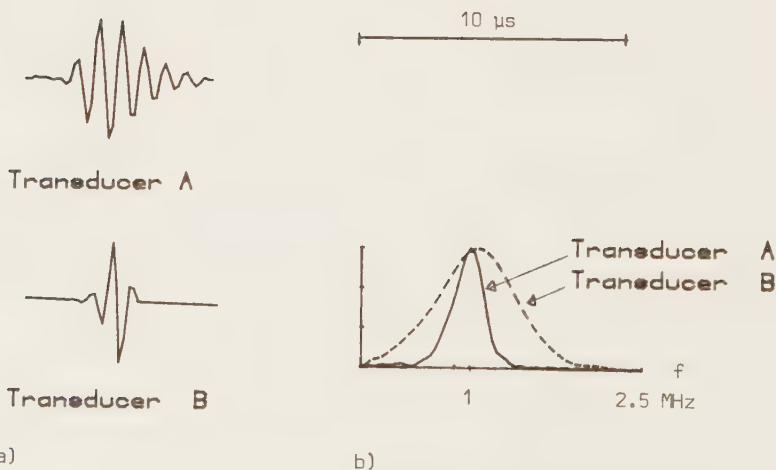


Figure 7. Examples of transducer impulse responses. Sample rate 5 MHz, signal length 40 points.

- a) The time signals.
- b) Their spectra (computed by a 128 points DFT).

of the impulse responses of these two transducers show significant differences in bandwidth. The centre frequency for both transducers is about 1 MHz and the sample rate has been 5 MHz.

The weighted least squares filter $u(t)$ calculated from the impulse response of transducer A is shown in Fig. 8a (in the frequency domain). The resolution interval is set to $T=4$ sampling points ($0.2 \cdot 4 \mu\text{s}$). Then the output pulse should have a duration of about $2 \cdot T = 8$ sampling points, i.e. $1.6 \mu\text{s}$. The output signal $y(t) = s(t) \cdot u(t)$ is shown in Fig. 8c and its spectrum in Fig. 8b. It is shown in Figs. 8b and 8c that the bandwidth has been increased and the duration decreased.

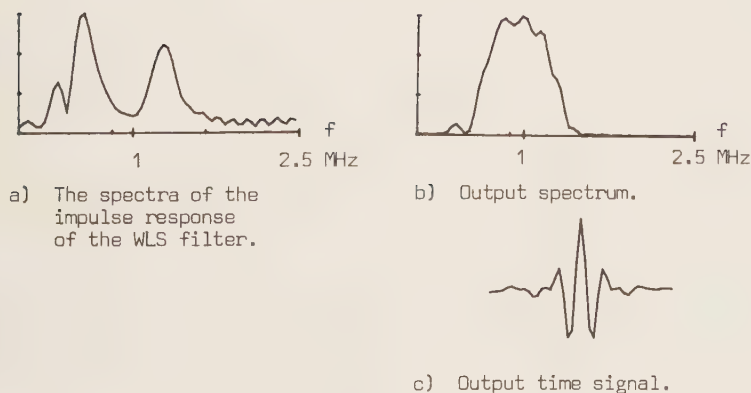


Figure 8. Example of the weighted least squares filter for transducer A (filter length equal to 40 points).

Examples of interfering echoes

It was shown in the previous example that the received pulse duration was decreased by using the weighted least squares filter. Examples of interfering echoes will now be presented. The test object is shown in Fig. 9. Two blocks of plexiglass are placed in a tank of water with a distance of 2 mm between the blocks as shown in Fig. 9. The blocks are 15 mm thick. The velocity of sound in water is approximately 1500 m/s. In plexiglass the velocity is about twice as high (3000 m/s). Then if the time axis is related to distance with respect to the velocity of sound in water, the thickness of the plexiglass blocks is incorrectly $15/2 = 7.5$ mm. The distance of 2 times 2 mm corresponds to 2.7 μ s.

The transducers A and B are tested against this target. This time the sample rate is set to 7.5 MHz. The signals are shown in Fig. 10. The first pulse in the received signal corresponds to the first water/plexiglass boundary, see Fig. 9. The second complex of pulses in the received signal corresponds to the boundaries around the 2 mm water gap.

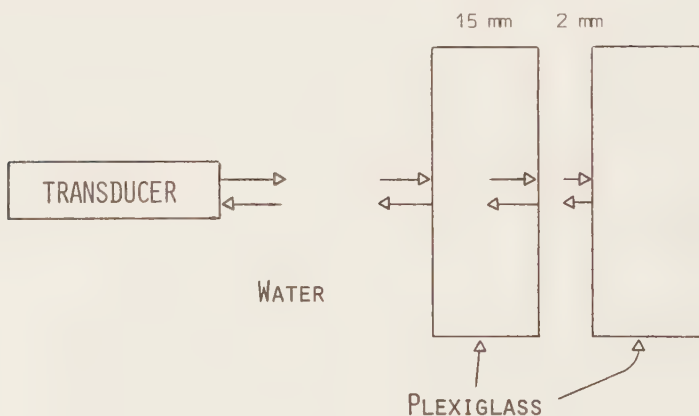
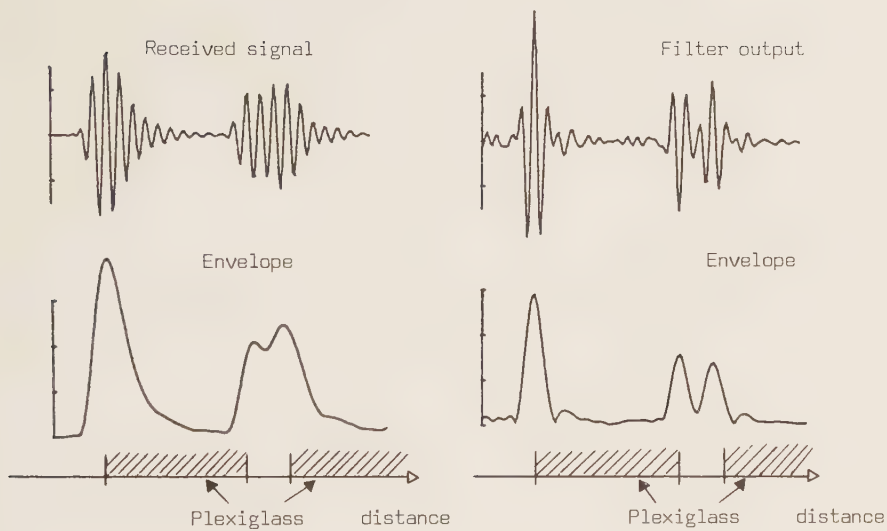


Figure 9. Test object for interfering echoes. The velocity of sound in water is about 1500 m/s and in plexiglass about 3000 m/s.

TRANSDUCER A



TRANSDUCER B

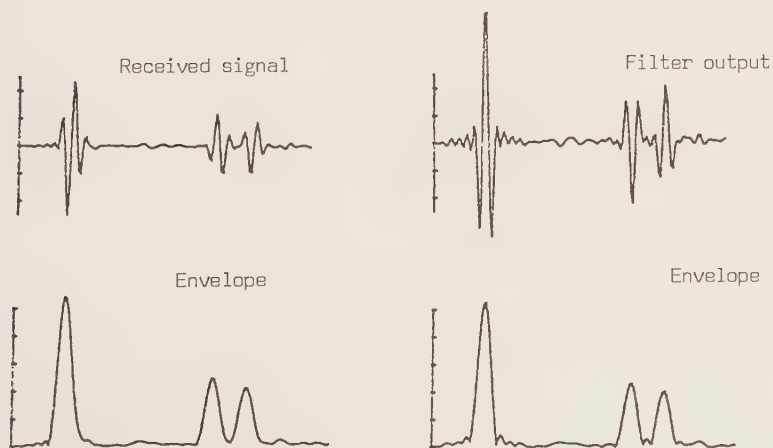


Figure 10. Examples of interfering echoes (sample rate 7.5 MHz).

In Fig. 10 the received signals and the filtered signals are shown for both transducers. The envelopes of these signals are also shown. Resolution has been significantly improved for transducer A. Transducer B is designed to give a short pulse and in this case only a small difference in the pulse duration from the filter output is achieved. In both cases the signs of the echoes are shown in the filter outputs.

Iterative solution

The weighted least squares filter (15) is the solution of a set of linear equations. But it can also be derived by an iterative procedure. A steepest descent algorithm for the solution of the filter is

$$\underline{u}^{k+1} = \underline{u}^k - \alpha \tilde{\underline{K}} \underline{e}^k \quad (16)$$

where the vector notations $\underline{u} = u(t)$ and $\underline{e} = e(t)$ are used. The matrix $\tilde{\underline{K}}$ depends on $s(t)$ and $w(t)$. The step size is given by α . The error signal is

$$e(t) = w(t) [s(t) * u(t) - \delta(t)] \quad (17)$$

The iterative solution is illustrated in Fig. 11. This solution scheme is used in the examples presented in the next section.

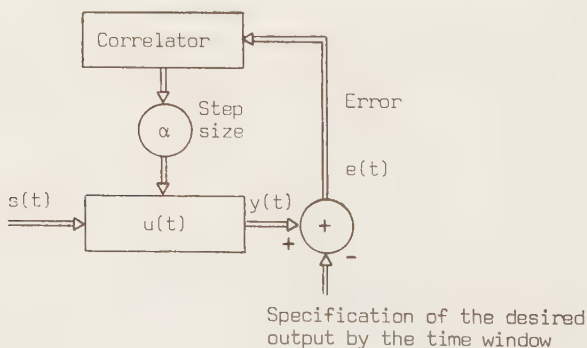


Figure 11. A block diagram of the iterative solution of the filter.

Band limited filters

The filter $u(t)$ is specially configured in the iterative scheme as a set of bandpass subfilters, i.e.

$$u(t) = \sum_k c_k q_k(t) \quad (18)$$

where $q_k(t)$ are bandpass subfilters and c_k the new parameters. With this configuration the rate of convergence of the steepest descent algorithm can be increased. The structure is described in [9]. For the remainder of this paper, this configuration is used for determining WLS filters. The set of bandpass subfilters in parallel is used to design time-limited filters which are also approximately limited to a selected frequency band. Figure 12 shows an example of a filter derived using (18). Transducer A is used and the selected frequency band is approximately 0.40 MHz to 1.60 MHz. The spectrum of the impulse response is shown in Fig. 12a and the impulse response in Fig. 12b.

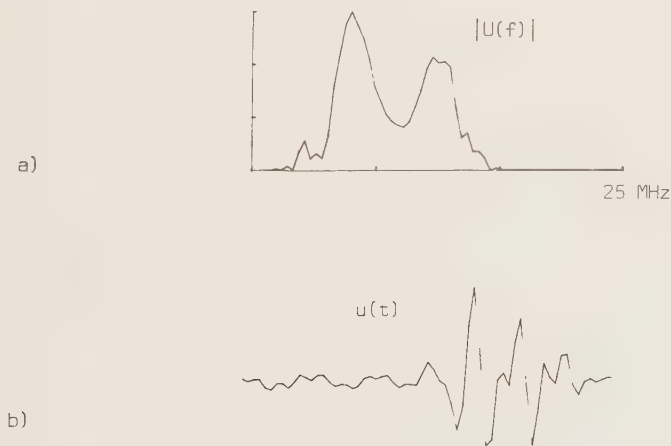


Figure 12. An example of the band limited filter.

a) The spectrum.

b) The impulse response of the filter, sample rate 5 MHz, filter length 64 points (12.8 μ s).

IV. PRESHAPING

It was shown in the previous section that the WLS filter applied to the received echo signal can improve the axial resolution. The calculations of the filtered echo signal were done off-line, because real-time digital processing at such high frequencies requires special-purpose hardware. A simple method suitable for on-line applications of the filtering technique is to preshape the transmitted ultrasonic pulse, i.e. to apply the filter to the transmitter rather than to the receiver, Fig. 13. The filter $u(t)$ that is used in the experiments reported here is the same WLS filter as was used previously to filter the received echo signal. These methods are equivalent in linear, time-invariant systems in the absence of noise.

The equipment used is described in [7]. The block diagram is shown in Fig. 14. The filter impulse response $u(t)$ is stored in a shift register. The signal $u(t)$ is triggered out of the shift register, passed through the D/A converter, amplified and emitted from the transducer as an ultrasonic pulse. The received echo signal is digitized in the A/D converter. The monitor in Fig. 14 shows both the excited waveform and the received echo signal.

Figure 15 shows the monitored echo signal received from the plexiglass target (Fig. 9) without preshaping. The lower trace is exciting signal, a rectangular pulse with the duration of 200 ns. The upper trace shows the received echoes. The first complex is from oscillations of the transducer. Then comes the echo from the first water/plexiglass junction, showing the impulse response of the transducer (transducer A described in Section III). The third complex represents interfering echoes from the surfaces around the 2 mm water gap. After that there are both multiple reflections and reflections from the far surface of the second plexiglass block. Figure 15 shows that the resolution of this transducer is not high enough to distinguish the surfaces around the water gap.

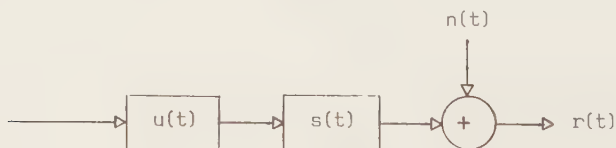


Figure 13. Preshaping of the transmitted ultrasonic signal.
Block diagram.

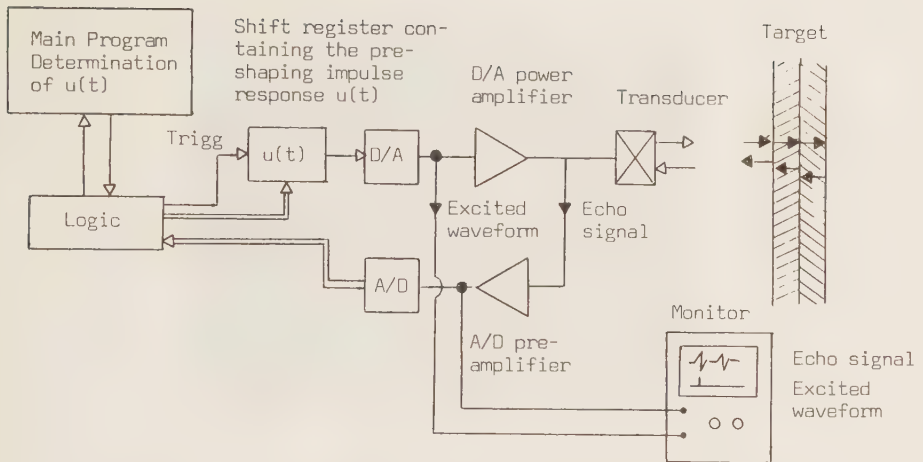


Figure 14. The block diagram of the experimental equipment used for preshaping the ultrasonic pulses.

In Fig. 16, the excited signal was the impulse response of the WLS filter with $T=6$ sample points (sample rate 5 Mz). The echoes are shown in the upper trace in Fig. 16. The resolution has now been improved so that the surfaces around the 2 mm water gap can be distinguished. Compare also with Fig. 10 where the filtering was on the received signal. The matched filter maximizes the signal/noise ratio but it unfortunately also increases the pulse duration. The echo signal when the matched filter impulse response excited the transducer is shown in Fig. 17. Further examples of the use of the WLS filter will be presented in next section.

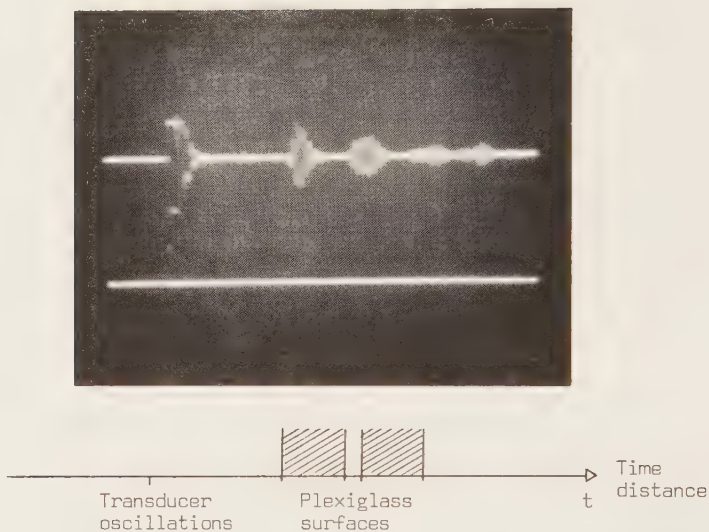


Figure 15. The monitored signal when the excited waveform was a rectangular pulse (transducer A). The lower trace shows the excited signal. The upper trace shows the received echoes.

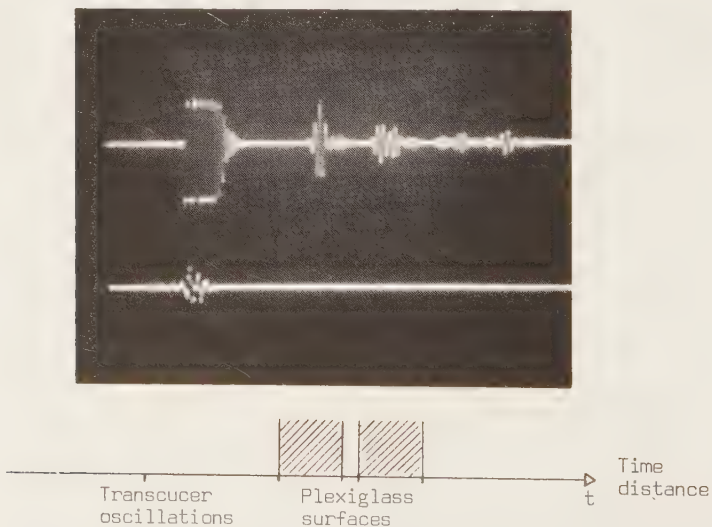


Figure 16. The monitored signals when the excited waveform was $u(t)$, the WLS filter impulse response (transducer A). The lower trace shows the excited signal. The upper trace shows the received echoes.

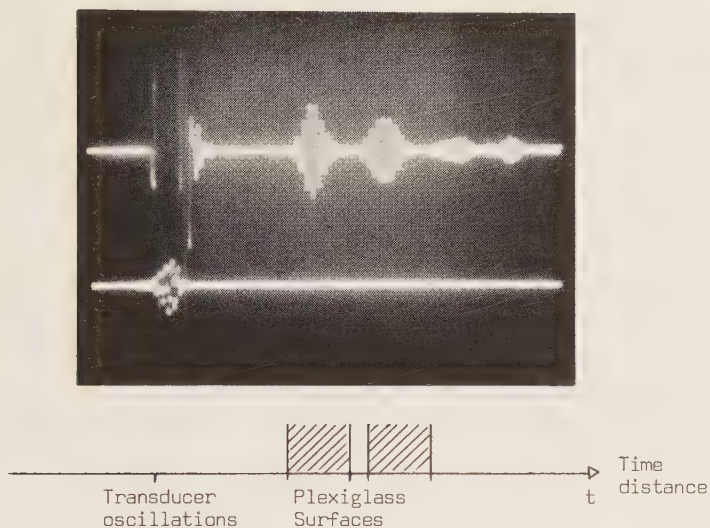


Figure 17. The monitored signal when the excited waveform was equal to the matched filter impulse response (transducer A). The lower trace shows the excited signal. The upper trace shows the received echoes.

V. EXPERIMENTS

This section will give some examples to demonstrate the improved resolution achieved by using the WLS preshaping filter. The aim is not to determine the performance or to optimize the filter (by varying the parameters T and w_0) but to show that this method can be used for various echo signals. The test equipment was influenced by nonlinearities introduced by the D/A power amplifier. These will not be taken into account in this paper but it is possible to compensate for some of them [7].

A transducer (Transducer C) with centre frequency of about 2 MHz was used here and the sample rate was 10 MHz. The impulse response and the spectrum are shown in Fig. 18. The WLS filter with $T=4$ sample points ($0.4 \mu s$) was used in this case as it was found to be most suitable. The value of w_0 is 0.1, corresponding to a signal/noise ratio of about 10 dB. The filter impulse response and its spectrum are shown in Fig. 19.

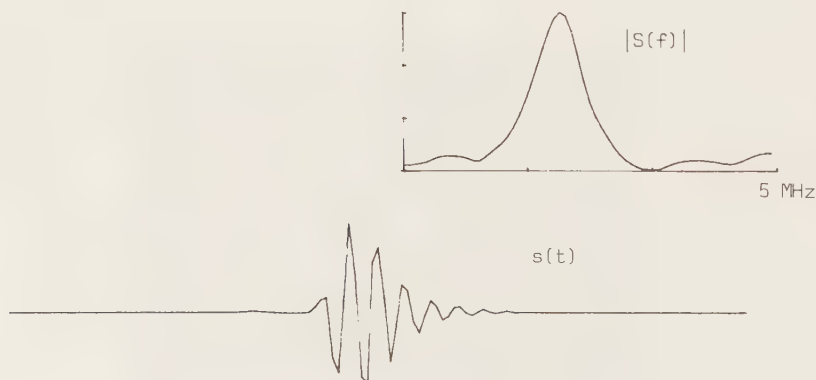


Figure 18. The impulse response $s(t)$ and the spectrum $S(f)$ of the transducer C, sample rate 10 MHz.

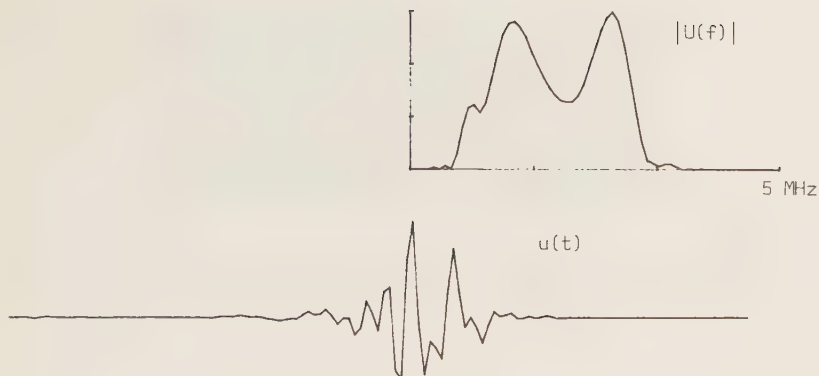


Figure 19. The WLS filter derived from $s(t)$ in Fig. 18. Frequency range about 0.7 to 3.0 MHz.

The plexiglass target

The echoes from the plexiglass target without preshaping are shown in Fig. 20. The echoes obtained when the WLS filter impulse response was used to excitate the transducer are shown in Fig. 21. The higher centre frequency of the transducer C also improves axial resolution so that the echoes from the surfaces around the 2 mm water gap can now be separated without preshaping. However, Fig. 21 demonstrates that resolution is improved by the WLS filter. Figures 20 and 21 show both the monitored analogue echoes and the digitized signal.

The lower frequency signal component in Figs. 20 and 21 and also in Figs. 22-24 comes from the radial oscillations of the transducer. These can be attenuated by inserting an inductance in the transducer cable. It is also apparent that these oscillations in the echo signals are smaller when the transmitted pulse is preshaped. This depends on the fact that the preshaping signal $u(t)$ contains fewer low frequency components, see Fig. 19.

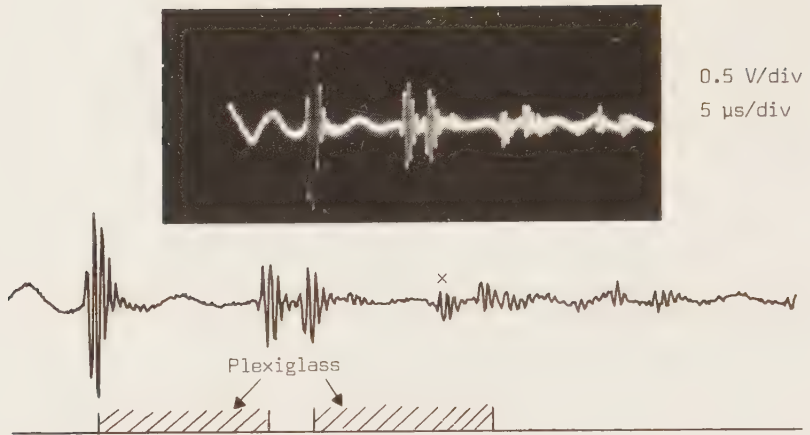


Figure 20. The monitored analogue echoes and the digitized signal obtained without preshaping, sample rate 10 MHz (transducer C, plexiglass target). The pulse marked x is an echo that has been reflected twice between the transducer and the plexiglass.

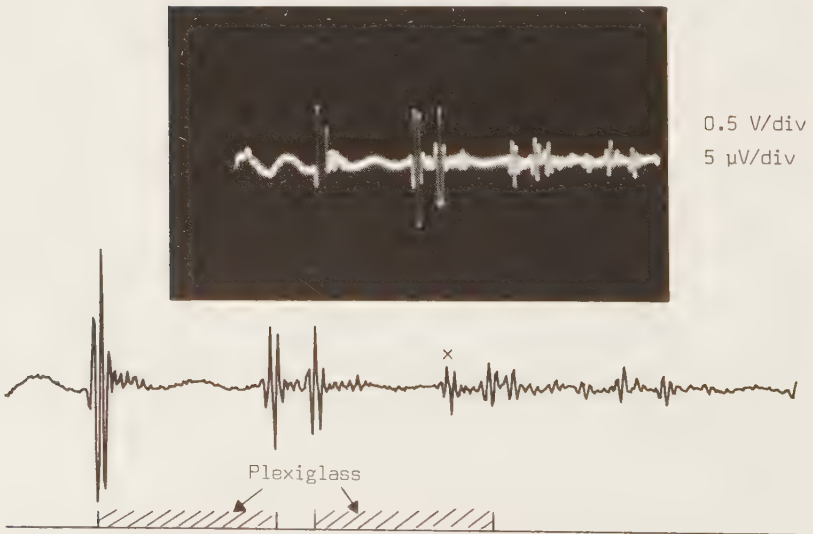


Figure 21. The monitored analogue echoes and the digitized signal obtained with preshaping by the impulse response of the WLS filter, sample rate 10 MHz (transducer C, plexiglass target), cf. Fig. 20.

Angled reflection surface

The transducer impulse response was measured from the echo from a surface perpendicular to the ultrasonic wave. The waveform of the echo will change if the surface is angled. In addition the amplitude of the echo then decreases and only echoes from surfaces with small variations of the angle will give echoes of high amplitude. Figure 22 shows an example where the transducer is angled so that the amplitude is only half of the maximum (cf. Fig. 21). Figure 22 shows a small variation in the waveform but resolution is still improved by the WLS filter.

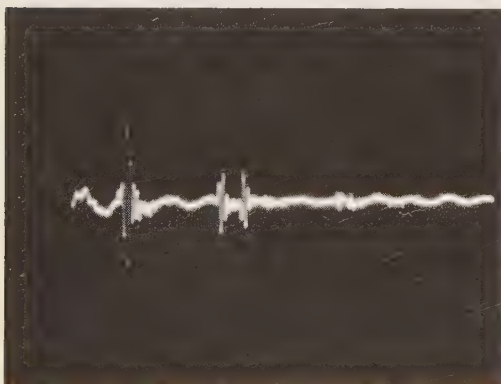
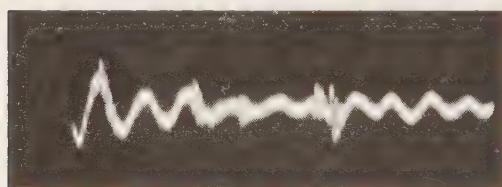


Figure 22. The echoes received from angled surfaces using the transducer C and the WLS filter (plexiglass target).

Echo signals from pork

Finally, examples of echoes from pork will be presented to demonstrate that the method is also suitable for complex echo structures. The echoes are quite small and the amplified echo signal rather noisy. The figures presented below do not show exactly the same part of the target as both the transducer and the target were held in the hand. Figure 23 shows the echoes received from a 20 mm thick piece of pork. The echoes to the right in Fig. 23 are from the rind. The thickness of the rind can be seen better in the preshaped signal.

An example from 40 mm thick piece of pork is shown in Fig. 24. The pre-shaped signal gives more distinct echoes. However, it is difficult to identify the reflection surfaces from one single sweep (trace) since the surfaces inside the pork are not perpendicular to the rind surface.

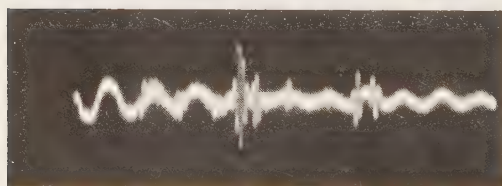


(No preshaping)

0.2 V/div

5 μ s/div

≈ 20 mm

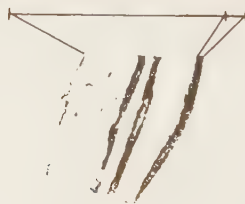


(Preshaping)

0.2 V/div

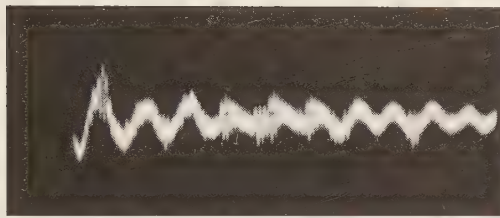
5 μ s/div

≈ 20 mm



Pork target

Figure 23. Echoes received from a pork target, with and without preshaping.



(No preshaping)

0.2 V/div

5 μ s/div

≈ 40 mm

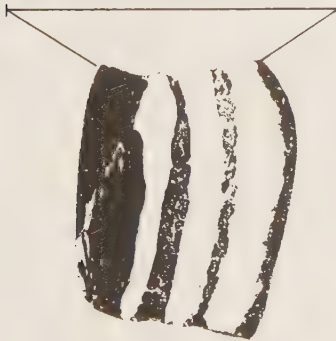


(Preshaping)

0.2 V/div

5 μ s/div

≈ 40 mm



Pork target

Figure 24. Echoes received from a pork target, with and without preshaping

VI. DISCUSSION AND CONCLUSIONS

The application of digital filtering techniques to ultrasonic echo signals has improved the axial resolution. The weighted least squares filter, specified by the resolution parameter T and the noise parameter w_0 , is suitable for ultrasonic signals. These parameters are "tuned" to give the filter its properties. Varying parameter T changes the filter to either an inverse filter or a matched filter (for small values of w_0).

This relationship of the weighted least squares filter to the matched filter indicates the robustness of the weighted least squares filter as it is used here. This is very important in practical applications. It is assumed here that the overall system is linear. In practical applications, however, the method must not fail if the system contains small nonlinearities. The attenuation of the ultrasonic signal as a function of distance is also frequency dependent. This is not taken into account in this paper, but the weighted least squares filter has shown such robustness that it can be used even in these cases, see Section V.

Implementation

Two of the used transducers (A and C) are moderately damped. It has been shown that the WLS filter makes an improvement of the axial resolution using these transducers. The transducer B is heavily damped to give a short duration of its impulse response and further shortening of its duration was difficult.

The application of digital filtering techniques to shaping ultrasonic signals should be taken into account in the manufacture of transducers. Transducers can now be manufactured with larger frequency ranges without necessity having to consider the duration of the impulse response. The desired shape of the impulse response will be given by the digital pre-shaping filter.

Further investigations

The preshaping filter described in Section IV applied to improving resolution permits real time processing with minor equipments. This could be implemented in existing equipment and it is planned to investigate this method further. The capability of the WLS filter to improve axial resolution need also further investigations. The influence of the nonlinearities in the test equipment (the D/A power amplifier) must be separated from the effect of the WLS filtering.

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